

GO Math! © 2015
A Research-Based Approach
September 2015



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GO Math! Efficacy Summary

This research base outlines the foundational research upon which the Houghton Mifflin Harcourt **GO Math!**[®] program was developed. The scientifically based research cited in this document provides evidence that **GO Math!** was designed using the latest academic and scholarly work about what constitutes good mathematics instruction.

Houghton Mifflin Harcourt™ **GO Math!** program's content and pedagogy were not only informed by academic research, but also direct efficacy research. For the past six years, numerous research studies have been performed to evaluate the program's effectiveness, and findings of this research have been used to improve and refine the instruction and structure of this program.

Early field tests of the program indicated that using the **GO Math!** approach was found to be significantly more effective at teaching young children critical mathematics skills and knowledge than other approaches typically used in classrooms.¹ Results from initial research were later replicated in longer studies of the program.

For instance, Educational Research Institute of America (2011) conducted a semester long study comparing students at Grades 3–5 using **GO Math!** to similar students using different mathematics programs. These researchers found that while students using **GO Math!** had similar mathematics performance at the beginning of the school year than comparison students, **GO Math!** students witnessed significantly more growth in math ability and were outperforming students by the end of the fall semester.²

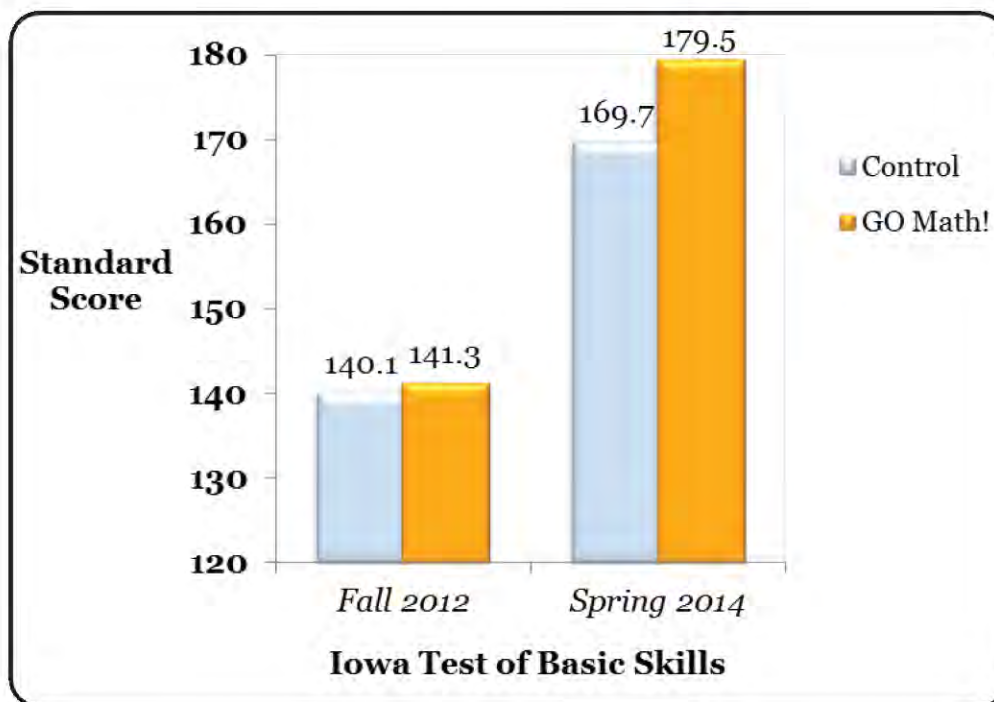
Growing research examining the impact of **GO Math!** on student standardized test scores has also revealed that implementation of the program is associated with marked increases in the percentage of students at or above proficiency levels on various state tests.³

The culminating effects of using **GO Math!** were recently revealed in a gold-standard, randomized control trial study of the program. This two-year study, conducted by independent research firm Cobblestone Applied Research and Evaluation, examined student math performance across 70 classrooms in seven states. Teachers were randomly assigned to either use **GO Math!** for the two school years, or continue using their existing curriculum. After two years of exposure to **GO Math!**, researchers found that students using the program had significantly greater state achievement scores as well as significantly higher scores on the Iowa Test of Basic Skills Form E, scoring an average of 6.7% higher than the control group.⁴ These effects were meaningful and conclusive, as this study design is the only research design that meets the What Works Clearinghouse's standards for program evaluation.⁵

For more information on the efficacy research for **GO Math!**, contact efficacy@hnhco.com

Figure 1

Results from a Two-year Study of **GO Math!**'s Impact on Students'
Iowa Test of Basic Skills Form E Performance³



Note: Unadjusted scaled scores. Significant differences ($p < .05$) were revealed using Multi-Level Modeling (MLM) controlling for student and teacher characteristics. Adopted from Cobblestone Research Applied Research & Evaluation (2014, December).

Footnotes

1. Educational Research Institute of America (2009, February). *A Control Group/Experimental Group Study of the Instructional Effectiveness of Houghton Mifflin Harcourt's Florida Math and Florida Strategic Intervention: Report 361*. Bloomington, IN: Author.
2. Educational Research Institute of America (2011, March). *A Study of the Instructional Effectiveness of GO Math! Report 399*. Bloomington, IN: Author.
3. Houghton Mifflin Harcourt (2015). *GO Math! Efficacy Research Booklet*. Boston, MA: Author.
4. Cobblestone Research Applied Research & Evaluation (2014, December). *Houghton Mifflin Harcourt GO Math! Efficacy Year Two Final Report*. La Verne, CA: Author.
5. For more information on the What Works Clearinghouse evidence standards, visit http://ies.ed.gov/ncee/wwc/pdf/reference_resources/wwc_procedures_v3_0_standards_handbook.pdf

Overview

Houghton Mifflin Harcourt's **GO Math!** © 2015 is a focused, comprehensive Kindergarten through Grade 6 mathematics program built for instruction on the Common Core State Standards. Developed around a coherent, focused progression across grade levels, **GO Math!** © 2015 is designed to help teachers as they support students through this rigorous curriculum. **GO Math!** © 2015 is part of a full K–12 curriculum that is built around the same principles outlined in the document.

The purpose of this document is to demonstrate clearly and explicitly the research upon which **GO Math!** © 2015 is based. This research report is organized by the major instructional strands that underpin the program:

- Effective mathematics teaching and learning;
- Instruction that meets the needs of all learners;
- Focused, coherent, and rigorous curriculum;
- Use of tools and technology to teach mathematics;
- Data-driven instruction and ongoing assessment; and
- Powerful forums for teacher improvement through professional learning.

Each strand is supported by research in math education and learning across content areas. The content, activities, and strategies presented in **GO Math!** © 2015 align with what we know about teaching for mathematical understanding and align to the Common Core State Standards for Mathematics.

To help readers make the connections between the research strands and the **GO Math!** © 2015 program, the following sections are used within each strand:

- **Defining the Strand.** This section summarizes the terminology and provides an overview of the research related to the strand.
- **Research that Guided the Development of GO Math!** This section identifies subtopics within each strand and provides excerpts from and summaries of relevant research on each subtopic.
- **From Research to Practice.** This section explains how the research data is exemplified in **GO Math!**

The combination of analytical recommendations and related features of **GO Math!** © 2015 will help readers better understand how the program incorporates research in its instructional design.

A list of sources is provided at the end of this document.

Introduction to GO Math! © 2015

Today's world is shaped by increasingly complex, dynamic, and powerful systems of information. Never before has the workplace demanded such complex levels of mathematical thinking and problem solving (National Council of Teachers of Mathematics, 2009). Those who understand and can do mathematics will have a wider range of opportunities, making it critical to build students' early foundational skills. An analysis of the results of the Trends in International Mathematics and Science Study (TIMSS) and the Program for International Student Assessment (PISA) led researchers to conclude that "countries that want to improve their mathematics performance should start by building a strong mathematics foundation in the early grades" (Ginsburg, Cooke, Leinwand, Noell, & Pollock, 2005, p. v).

The Common Core State Standards for Mathematics were written to provide this exact foundation for young students. The standards are focused, coherent, and rigorous—and describe the content and skills needed for young students to "build the foundation to successfully apply more demanding math concepts and procedures, and move into applications" (Common Core State Standards Initiative, 2011).

The Common Core State Standards (CCSS) were systematically developed to:

- Be research-based;
- Focus on the critical skills at each grade level;
- Encourage conceptual mastery of key ideas;
- Develop students' mathematical understanding and procedural skill and fluency;
- Build students' ability to apply math flexibly in context;
- Present a coherent progression from grade to grade; and
- Prepare students for the demands of the future—in school and work.

In addition, the Standards serve the purpose of helping to ensure equity for all American students. Inconsistent standards, curriculum, and assessments across states have raised equity issues in the past (Reed, 2009) and wide disparities in performance on the National Assessment of Educational Progress (NAEP) (Schneider, 2007).

While the standards detail the knowledge and skills—content and processes—students need at each grade level, they do not describe the instructional approaches needed to meet the standards. Thus, an effective instructional program is needed to bridge between the expectations set out by the standards and the desired student learning and achievement.

This alignment between standards, curriculum, instruction, and assessments is critical. Researchers looking at effective educational practices identified nine characteristics of high-performing schools and reported that several of these relate to standards and standards alignment. High-performing schools have a clear, shared focus; high standards and expectations for all students; and curriculum, instruction, and assessments aligned to the standards (Shannon & Bylsma, 2003).

Houghton Mifflin Harcourt's **GO Math!** © 2015 was developed with the Common Core State Standards for Mathematics as a foundation and uses research-tested approaches. **GO Math!** © 2015 is a program that is:

Focused—Content is focused on essential learning so that students have time to master content at each grade level. The grade level CCSS Critical Area organization of **GO Math!** © 2015 focuses on key big ideas, while chapters align to domains and standard clusters to build connections among the individual standards.

Coherent—Content is organized into meaningful progressions that seamlessly connect key topics between the grade levels of K–12. **GO Math!** utilizes a related set of mathematical models, problem types, and instructional strategies to support a cohesive learning path within, and between, all grade levels. Lessons include coherently sequenced learning experiences to develop critical understandings. This approach facilitates connections between major topics.

Rigorous—Content is presented for students to develop a deeper understanding. **GO Math!** © 2015 guides teachers to teach for depth and supports students to build understanding, fluency, and applications to problem solving.

Throughout **GO Math!** © 2015, alignment with the Common Core is made explicit, with standards and mathematical practices references included alongside lesson content and in the program's table of contents.

Strand 1: Teaching and Learning

An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.

(NCTM Principles to Actions, 2014, p. 7)

Defining the Strand

Quality teaching matters. Extensive research has shown that teacher effectiveness has an enormous impact on student learning and achievement—more than any other in-school factor (Goldhaber, 2002; Partnership for Learning, 2010). Chetty, Friedman, and Rockoff (2012) looked at the long-term impacts of teachers and found that those who added value to their students' test scores also added life-long value to their students' educational attainment and income earning.

What is it that effective teachers do to make gains in student learning and achievement? Quality teachers use effective classroom practices (Wenglinsky, 2002). Research—in cognitive science, on classroom practices of master teachers, and on specific supports that help students learn—points to specific principles and methods of effective instruction (Rosenshine, 2012). Teaching mathematics is not easy, but employing proven research techniques can help teachers ensure all students learn.

GO Math! © 2015 is a program designed to support teachers in effectively building students' mathematical skills and understandings. In its design, the program incorporates research-based strategies for effective teaching to maximize student learning. **GO Math!** supports teaching and learning by incorporating the following instructional approaches:

- Using the 5E Instructional Model;
- Establishing clear mathematical goals;
- Incorporating tasks that promote reasoning and problem solving;
- Using and connecting mathematical models and representations;
- Communicating mathematically;
- Presenting purposeful questions;
- Building procedural fluency from conceptual understanding;
- Supporting students' mathematical struggles; and
- Eliciting and using student work to gauge understanding.

Research that Guided the Development of the Go Math! Program

5E Instructional Model (Engage, Explore, Explain, Elaborate, Evaluate)

With roots to historical models dated back to the 1900s, the 5E Instructional Model was developed through the Biological Sciences and Curriculum Studies (BSCS) in the late 1980s and is grounded in decades of solid, effective research. The 5E Instructional Model offers a predictable, structured, research-based sequence consisting of five phases: engagement, exploration, explanation, elaboration, and evaluation, which are briefly described below (Bybee et al., 2006).

1. Engagement: Students access prior knowledge through teaching and tasks to help them become engaged in the concept.
2. Exploration: Students generate new ideas and explore possibilities.
3. Explanation: Students demonstrate understanding through explaining what they know, giving the teacher an opportunity to guide them toward deeper learning.
4. Elaboration: Students extend and apply their knowledge.
5. Evaluation: Students examine their own understanding, while the teacher continues to monitor and evaluate progress.

In the National Research Council's 1999 publication called *How People Learn*, research on learning was synthesized and findings supported the core tenets of the 5E Instructional Model. "An alternative to simply progressing through a series of exercises that derive from a scope and sequence chart is to expose students to the major patterns of a subject domain as they arise naturally in problem situations. Activities can be structured so that students are able to explore, explain, extend, and evaluate their progress. Ideas are best introduced when students see a need or a reason for their use—this helps them to see relevant uses of the knowledge to make sense of what they are learning" (NRC, 1999, p. 127).

In support of the NRC findings, recent research by Coulson (2002) found that students whose teachers taught with medium or high levels of fidelity to the 5Es experienced learning gains that were nearly double that of students whose teachers implemented the model with a lower level of fidelity or did not implement it at all. As well, a study by Boddy, Watson, and Aubusson (2003) noted that students whose teachers used the 5Es showed increases in scientific reasoning.

Establishing Mathematical Goals

Identifying and clarifying what students are expected to learn and understand in a mathematics classroom is an essential component to success (William, 2011). Teachers and students benefit from establishing a shared foundation of what is being learned and why it is important to learn. "Formulating clear, explicit learning goals sets the stage for everything else" (Hiebert, et al., 2007, p. 57).

Learning goals must also be positioned and clearly articulated within the larger mathematical trajectory. By looking at the goals within mathematics learning progressions (Charles, 2005), teachers have the opportunity to examine and monitor student progress and needs in order to adjust instruction as necessary (Clements & Sarama, 2004; Sztajn et al., 2012). Teachers can support students as they build

on what they know, develop more complex understandings, and realize that mathematics is not a set of discrete parts—it is coherent and connected (Fosnot & Jacob, 2010; Ma, 2010).

Research also shows that setting and sharing learning goals with students has a positive impact on their learning. Work by Haystead & Marzano (2009) and Hattie (2009) shows that students in classrooms where learning goals are clearly articulated perform at higher levels than students who are unaware of the expectations. When expectations are discussed with students, they are able to find value in their work and understand the greater purpose of what they are learning (Black & Wiliam, 1998a; Marzano, 2009). Establishing goals allows students to focus on the expectation that is set and become more aware of their own thinking and learning (Clarke, Timperley, & Hattie, 2004; Zimmerman, 2001).

Incorporating Tasks that Promote Reasoning and Problem Solving

There is no decision that teachers make that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the development of the tasks with which the teacher engages students in studying mathematics (Lappan & Briars, 1995). What types of tasks should students practice? Tasks that allow students to make connections based on what they know, explore real-world problems, and promote higher-level thinking.

Connections—among mathematical ideas, with other content areas, and in real-world contexts—are an essential part of mathematics learning. Making connections between new information and students' existing knowledge—knowledge of other content areas and of the real world—has proved to be more effective than learning facts in isolation (Beane, 1997; Bransford, Brown, & Cocoking, 1999; Caine & Caine, 1994; Kovalik, 1994). Further, connecting mathematics to science, social studies, and business topics can increase students' understanding of and ability with mathematics (Russo, Hecht, Burghardt, Hacker, & Saxman, 2011). Students maximize learning when they can make connections between ideas.

Students see the purpose and value of learning when they experience it in real-world contexts. "When instruction is anchored in the context of each learner's world, students are more likely to take ownership for...their own learning" (McREL, 2010, p. 7). Connecting to the tasks improves their perception of the content as interesting and beneficial, thereby increasing their motivation to learn (Czerniak, Weber, Sandmann, & Ahem, 1999). Students learn best when they can understand the purpose and relevance of what they are learning in relation to the larger world around them.

Encouraging and supporting students' tasks that require a high level of cognitive demand is necessary when promoting reasoning and problem solving in the mathematics classroom. In a study that compared students exposed to teaching strategies that promoted higher-order thinking with those who were taught more traditionally, researchers found that experimental group students outperformed control group students, showing significant improvement in their critical thinking skills; "Our findings suggest that if teachers purposefully and persistently practice higher order thinking strategies, for example dealing in class with real-world problems, encouraging open-ended class discussions, and fostering inquiry-oriented experiments, there is a good chance for a consequent development of critical thinking capabilities" (Miri, David, & Uri, 2007, p. 353). Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural in nature. (Boaler & Staples, 2008; Stein & Lane, 1996).

Using and Connecting Mathematical Models and Representations

Representations and models include a wide variety of images or likenesses that can be used to support and extend understanding in the mathematics classroom. Representations and models might include pictures, manipulatives, symbols, diagrams, or organizers that students create or teachers provide to show a mathematical construct or action.

According to NCTM, “Representations should be treated as essential elements in supporting students’ understanding of mathematical concepts and relationships; in communicating mathematical approaches, arguments, and understandings to one’s self and to others; in recognizing connections among related mathematical concepts; and in applying mathematics to realistic problem situations through modeling” (NCTM, 2000, p. 67). Essentially, representations can show what students know, help students explain what they know, and be the foundation for making connections and achieving a deeper understanding of mathematics.

Students’ use of mathematical models and representations can help to make mathematical concepts more concrete to students. “Because of the abstract nature of mathematics, people have access to mathematical ideas only through the representations of those ideas” (NRC, 2001, p. 94). For young students, representations are especially important because they can be physical objects or actions they perform as they are trying to solve problems (NRC, 2009). As students create representations, they have the opportunity to internalize and process what they are doing, what they are creating, and what they are seeing, which in turn allows them an active role in their learning.

To illustrate this point, a recent study indicates that using visual representations has shown to improve student performance in general mathematics, prealgebra, word problems, and operations (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009). When students sketch or organize their mathematical thinking, they are able to explore their understanding of concepts, procedures, and processes—and communicate mathematically (Arcavi, 2003; Stylianou & Silver, 2004). This is especially beneficial to students who have special needs, struggle with learning, or are English language learners. Having students participate in discussions about their representations allows for meaningful learning (Fuson & Murata, 2007).

Representations also impact learners’ success in changing how they think. When examining multiple representations, students can decide how to best solve problems and practice looking at problems from different points of view. Moving between and among representations improves the expansion of students’ ability to understand concepts (Lesh, Post & Behr, 1987).

Communicating Mathematically

According to Carpenter, Franke, and Levi (2003), “students who learn to articulate and justify their own mathematical ideas, reason through their own and others’ mathematical explanations, and provide a rationale for their answers develop a deep understanding that is critical to their future success in mathematics.” Mathematical discourse is an essential practice in the mathematics classroom.

Communicating mathematically is a consistent thread throughout research on effective instructional strategies for teaching mathematics—and is one of the strategies highlighted in NCTM’s *Principles and*

Standards for School Mathematics and is a recurrent thread throughout the Common Core Standards for Mathematical Practice. When students write about and discuss math concepts, they have the chance to think through, defend, and support their ideas. A review of studies conducted by the National Council of Teachers of Mathematics revealed that “the process of encouraging students to verbalize their thinking—by talking, writing, or drawing the steps they used in solving a problem—was consistently effective... Results of these studies were quite impressive, with an average effect size of 0.98...” (Gersten & Clarke, 2007, p. 2). Communicating about math improves learning; “encouraging students to verbalize their current understandings and providing feedback to the student increases learning” (Gersten & Chard, 2001, online).

Numerous studies have emphasized the importance of writing in the mathematics classroom. Bosse and Faulconer (2008) report that writing in the mathematics classroom results in deeper student learning. As noted by several researchers, conceptual understanding and problem-solving skills improve when students are encouraged to write about their mathematical thinking (Burns, 2004; Putnam, 2003; Russek, 1998; Williams, 2003). Writing during math instruction has been found to give students more confidence in their math abilities, create more positive attitudes toward math, and helps students to understand complex math concepts (Furner & Duffy, 2002; Taylor & McDonald, 2007). Importantly, writing appears to benefit all students, with researchers finding benefits for low-achieving students (Baxter, Woodward, & Olson, 2005) and for high-achieving students (Brandenburg, 2002).

Writing can be incorporated into the mathematics classroom in numerous ways, including free writing, biography, learning logs, blogs, journals, summaries, word problems, and formal writing (Urquhart, 2009). Students can engage in more structured or more informal journaling or note taking. In a study with Grade 9 algebra students, Pugalee (2004) found that journal writing positively impacted students’ problem solving. Albert and Antos (2000) examined the impact of journal writing, and found that using the journals “gives students practice in communicating their ideas clearly and allows for each student to make a personal connection that strengthens his or her learning and understanding of mathematical concepts and ideas” (p. 530–531). In addition to writing, verbalizing is an effective method of developing students’ grasp of mathematics.

Providing opportunities for students to talk about mathematics and mathematical concepts also enhances their understanding of mathematics. Instructional practices—such as restating, prompting students, and engaging in whole-class discussion, small-group discussion, and paired conversations—have been shown to be effective in improving student understanding (Chapin, O’Connor, & Canavan Anderson, 2003). Hatano and Inagaki (1991) found that students who discussed and justified their solutions with peers demonstrated greater mathematical understanding than students who did not engage in such discussions. Leinwand and Fleischman (2004) reviewed research on effective mathematics instruction and concluded that talking about math and explaining the rationale for solutions can help ensure that students have a conceptual understanding.

Talking about math has also been found to benefit students at different levels of learning and in different contexts. In their study, Hufferd-Ackles, Fuson, and Sherin (2004) found a math-talk community to be beneficial with students who were English language learners in an urban setting. Similarly, working in a transitional language classroom led researchers Bray, Dixon, and Martinez (2006) to conclude that as students “communicate verbally and in writing about their mathematical ideas, they not only reflect on and clarify those ideas but also begin to become a community of learners” (p. 138).

In addition to promoting greater learning, communication in the mathematics classroom can facilitate teachers in assessing students' performance—and students in engaging in self-assessment; "Classroom communication about students' mathematical thinking greatly facilitates both teacher and student assessment of learning" (Donovan & Bransford, 2005, p. 239).

Understanding the language of mathematics is essential to understanding and doing mathematics. In numerous studies, students' knowledge of mathematical vocabulary has been shown to correlate with their mathematics achievement (Earp, 1970; Stahl & Fairbanks, 1986; Usiskin, 1996). Research by Freeman and Crawford (2008) found that focused and explicit attention to vocabulary and language helps students develop a deeper understanding of content.

Why is vocabulary learning so tied to mathematics achievement? This may be because the language of mathematics is so closely tied to the content of mathematics; mathematics vocabulary terms offer the means to communicate mathematical ideas that are by nature abstract and complex (Kouba, 1989).

To support students' proficiency, teachers should incorporate vocabulary instruction in the math classroom so that students can understand and use this "language with its own symbols, syntax, and grammar" (Leiva, 2007, online). Certain strategies have been shown to be particularly effective. Research suggests that instructional practices for promoting vocabulary learning must include *integrating* new vocabulary, connecting it to previously learned words and concepts; *repeating* vocabulary words and offering opportunities for practice with new words; and *using words with meaning*, or offering opportunities for students to use the words in meaningful ways (Harmon, Hedrick, & Wood, 2005).

While instruction in vocabulary in mathematics benefits all learners, explicit instruction in the language of mathematics is particularly important for English language learners and for struggling readers (Bay-Williams & Livers, 2009).

Presenting Purposeful Questions

"Effective teaching of mathematics uses purposeful questions to assess and advance student reasoning and sense making about important mathematical ideas and relationships" (NCTM, 2014). While types of questions vary from asking students to recall facts to requesting an explanation for an answer, presenting questions throughout the learning process is necessary to understand how students are making sense of the math. As noted in research by Weiss and Pasley (2004), questions are critical in helping students make connections and learn important mathematics concepts—especially questioning that effectively gauges student understanding. In a study of over 364 lessons, Weiss and Pasley noted "evidence of a culture conducive to learning when teachers asked questions that challenged and broadened students' thinking" (p. 24). In contrast, as noted in the study *Inside the Classroom*, Weiss et al. observed "...lessons judged to be low in quality are characterized by learning environments that are lacking in respect and/or rigor; questioning that emphasizes getting the right answer and moving on, without also focusing on student understanding..." (p. 104).

Building Procedural Fluency from Conceptual Understanding

To achieve understanding, students need instruction that recognizes the relationship between procedural fluency and conceptual understanding. Specifically, “Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems” (NCTM, 2014, p. 42). Effective mathematics instruction cannot have one without the other as “procedural knowledge and conceptual understandings must be closely linked” (NRC, 2005, p. 232).

A wide body of reports recognizes the critical relationship and balance between concepts and procedures in mathematics instruction for student learning (National Mathematics Advisory Panel, 2008; National Research Council, 2001). Also evidenced by results from a study conducted by Rittle-Johnson & Alibali (1999), concepts and procedures develop iteratively—and gains in one area lead to gains in the other.

Research by Hiebert suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). When learning is not meaningful and disconnected, students have a more difficult time absorbing concepts. When students are able to connect procedures and concepts, their retention improves and they are better able to apply what they know in different situations (Fuson, Kalchman, & Bransford, 2005).

All students need to have a deep and flexible knowledge of a variety of procedures, along with an ability to make critical judgments about which procedures or strategies are appropriate for use in particular situations for best success in the mathematics classroom (NRC, 2001, 2005, 2012; Star, 2005).

Supporting Students’ Mathematical Struggles

When providing mathematics instruction, it is beneficial for teachers to allow students to explore what they know—and what they don’t know yet. Allowing students the opportunity to practice and struggle with mathematical problems and ideas encourages them to think about their own thinking and to discover that learning can happen without rushing to simply find the correct answer. Teachers who guide students through “productive struggle” are supporting the development of student learning and understanding (Hiebert & Grouws, 2007). Students receive reassurance as they grapple with ideas, and teachers support them through the process rather than give them the answers (Hiebert et al, 1996).

In his research on “Productive Failure,” Kapur (2010) found that students given time to make mistakes and persist through their struggles ultimately showed greater understanding on post-test measures than their counterparts. Kapur’s research supports the findings of two previous studies (Dillenbourg, 2002; Scardamalia & Bereiter 2003), which also address the value in allowing students to experience struggles in learning.

While fostering an environment where students are free to work through difficulties, as instructional guides, teachers need to keep students connected to the learning process. Without positive guidance and reinforcement, a “productive struggle” could become an “unproductive struggle,” in which students “make no progress towards sense-making, explaining, or proceeding with a problem or task at hand” (Warshauer, 2011, p. 21).

Research has also shown that learners are engaged in the learning process when they are asked to explain and reflect on their thinking processes (Good & Whang, 1999; Hettich, 1976; Surbeck, 1994). Continuing to work with students, asking them to explain and justify how they solved problems helps to maintain student engagement (National Research Council, 2001).

Other research points to scaffolding as a strategy to keep students connected to learning. Scaffolding can help students to believe in their own abilities to succeed (Baker, Schirner, & Hoffman, 2006). As Hyde (2006) states, “Scaffolding does not necessarily make the problem easier, and the teacher does not do the work for students or show them how to do it. It enables the person to do it” (p. 28). This empowerment gives students confidence in their ability and allows them to take on increasingly more challenging material and assignments as they demonstrate success completing previous tasks. Williams (2008) found that “scaffolding tasks allowed students to work independently at appropriately challenging levels...and develop a sense of self-confidence in their mathematics knowledge and skills” (p. 329).

To be motivated to learn, students must have the expectation that they can learn and a belief that their learning has value (National Research Council, 2001). Research has long documented the connection between a student’s sense of confidence and self-efficacy for learning and his or her learning and achievement. In their study, in which they investigated the relationship between achievement and learning in mathematics and motivational and affective variables, Seegers and Boekaerts (1993) found that cognitive levels alone did not explain differences in mathematical performance. Rather, students’ self-efficacy, their perceptions of the relevance and interest of tasks, and their willingness to invest effort all contribute to their performance. Students who believe they can learn persist in learning and are engaged in learning, and subsequently learn more than peers who are less confident in their abilities.

Eliciting and Using Student Work

To discover what students know or don’t know, what they do well or do poorly, the teacher must closely examine the students’ work. “Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning” (NCTM, 2014, p. 53).

When incorporated into classroom instruction, collecting and reviewing student work is a powerful tool in addressing—and preventing—student misconceptions. After a review of research on the principles of effective instruction, Rosenshine (2012) came to the conclusion that one of the most important elements of effective instruction is that the teacher continually checks for student understanding—in part to determine if students held previous misconceptions, or were developing new misconceptions. Additional research suggests that failing to provide feedback not only fails to challenge students’ existing misconceptions, but may also enable students to create new misconceptions (Brown & Campione, 1994). Examining student evidence throughout the instructional process affords teachers the chance to work with students’ misunderstandings before they persist.

Examining incorrect student responses can “reveal specific student misunderstandings” (Popham, 2006, p. 86)—and teachers can respond instructionally. By analyzing the mistakes that students make, teachers can determine which specific concepts, algorithms, or procedures need additional instruction (Ketterlin-Geller & Yovanoff, 2009). Studying student responses as a group can also allow teachers to find evidence of any recurring misconceptions held by several students (NCTM, 2000).

From Research to Practice

5E Instructional Model in *GO Math!*

GO Math! © 2015 follows the principles of an effective instructional process that supports all students in learning. The entire program was conceptualized for, organized around, and aligned to the Common Core State Standards. The program focuses on how best to support students in meeting these high expectations.

GO Math! follows a regular, predictable, five-step instructional model:

1. **Engage**—the **Essential Question** gets students thinking about the focus of the lesson, while the **Making Connections** suggestions help to activate prior knowledge to get students interested and connected to the content.
2. **Explore**—activities and exercises that encourage students to investigate lesson content and practice skills are provided. **Listen and Draw** (Grades K–2) and **Unlock the Problem** (Grades 3–6) introduce students to real-world situations and applications while also supporting the Standards for Mathematical Practices.
3. **Explain**—students engage in activities like the following to explain their understandings—**Math Talk, Share and Show**, and **Talk About It**. Teachers have access to numerous suggestions to help students **Go Deeper**.
4. **Elaborate**—students elaborate on their understandings of content and new skills through discussions, through practice (using print and digital activities and tools), and through **Problem Solving Applications**. Teachers can make use of the many opportunities to support students to **Think Smarter**, including using the **Math on the Spot** videos available to students in the Student Edition.
5. **Evaluate**—teachers assess student knowledge and skills using the formative assessment options available. Teachers work to understand what students know about the **Essential Question** and gauge understanding through questions, discussions, **Lesson Checks**, **Math Journals**, and **Practice and Homework**.

The program supports teachers through every stage of the instructional planning process helping them with planning tools that include the following:

- The **Teacher Digital Management Center** offers access to a full suite of teaching resources online, where teachers can plan, present, and manage classes, assignments, and activities. mySmartPlanner helps teachers customize lesson plans, create and view assignments, and access all program resources in one location.
- The **Teacher's Edition** helps teachers support students with point-of-use questioning strategies, teaching tips, tools for differentiated instruction, additional activities, and more.
- The **Chapter Resource Books** (one for each chapter), contain Family Letters, Reteach and Enrichment Worksheets, and Assessments.

Establishing Mathematical Goals in *GO Math!*

Throughout the *GO Math!* © 2015 program, the mathematical goals are clear at the onset of each lesson and are reinforced throughout.

Prior to each lesson is a **Lesson at a Glance** chart, which clearly outlines the content focus, how the content applies across the grade levels, and how to support the rigor throughout the lesson. Related Common Core State Standards and Mathematical Practices are also outlined.

FOCUS **COHERENCE** **RIGOR**

LESSON AT A GLANCE

FCR Focus:
 **Common Core State Standards**
5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
MATHEMATICAL PRACTICES
MP4 Model with mathematics **MP5** Use appropriate tools strategically.
MP7 Look for and make use of structure.

FCR Coherence:
Standards Across the Grades
Before Grade 5 After
4.NBT.A.1 5.NBT.A.1 6.NS.B.3
4.NBT.A.2
4.NBT.A.3

FCR Rigor:
Level 1: Understand Concepts.....*Share and Show*  **Checked Items**
Level 2: Procedural Skills and Fluency.....*On Your Own*
Level 3: Applications.....*Think Smarter and Go Deeper*

Learning Objective
Model, read, and write decimals to thousandths.

Language Objective
Student partners describe the relationship between two decimal place-value positions to another pair.

Materials
MathBoard, color pencils, straightedge

FCR For more about how *GO Math!* fosters **Coherence** within the Content Standards and Mathematical Progressions for this chapter, see page 149!

Each lesson begins by introducing the **Essential Question**. Teachers and students kick off each lesson with a shared understanding of the goal, either by using the Interactive Student Edition or through teacher-led discussions prompted in the Teacher Edition. Activating prior knowledge and understanding helps to support moving forward with the lesson.

1 ENGAGE

with the Interactive Student Edition

Essential Question

How do you model taking from a group?

Making Connections

You may want to hand out ten cubes and review how the children have used cubes for counting and addition.

Let's use cubes to show an addition story about dogs. There are 5 dogs. How many cubes will you use to show 5 dogs? 5 cubes Now 3 dogs join the group. How many cubes will you use to show the 3 dogs? 3 cubes How many dogs do you have altogether? 8 dogs

In *GO Math!*, the **Essential Question** is reinforced during the evaluation stage of the instructional model as well. As evidenced by these sections, the focus for each lesson and each chapter is clearly articulated and maintained.

Incorporating Tasks that Promote Reasoning and Problem Solving in *GO Math!*

In *GO Math!* © 2015, the tasks are carefully crafted to promote reasoning and problem solving, which also support the rigor reflected in the Common Core State Standards. Reasoning and problem solving are reinforced throughout the chapters and lessons in *GO Math!* in a variety of ways.

Real-World Projects—these projects are designed to help introduce each Critical Area. The projects are engaging, thought provoking, and require students to apply what they know.

INTRODUCE THE PROJECT

Multiplication and division are used every day in real-world situations. Have students look at the photograph. Explain that the astronauts on the space shuttle must carry with them all the food and water they need while in space. People who plan the space shuttle missions use multiplication to calculate how many pounds or gallons of different supplies the astronauts will need.

Problem Solving Applications—each lesson includes specific references to the use of mathematical practices to make sense of real-world tasks.

Problem Solving • Applications

MATHEMATICAL PRACTICES

MP4 Model with mathematics. Have students read Exercise 11. Ask them to describe how they will solve the problem.

Go Deeper and **Think Smarter**—each lesson includes these problems and tasks that are specifically designed to develop higher-order thinking skills, prepare students for the Common Core assessment, and promote rigor in mathematics.

GO DEEPER

For Exercise 16, suggest students think of the expressions as 2 groups of 6 and 6 groups of 2.

THINK SMARTER

Exercise 17 requires students to interpret information and complete multiple steps to find the total.



**Math on the Spot
Video Tutor**

Use this video to help students model and solve this type of *Think Smarter* problem.



Math on the Spot videos are in the Interactive Student Edition and at www.thinkcentral.com.

THINK SMARTER

Students should recognize the application of the Commutative Property of Multiplication using a different order of the same factors. If students do not understand that factors may be multiplied in any order and the result will be the same, they will make at least two incorrect connections.

Other resources that support the reasoning and problem solving include:

- **Listen and Draw** (Grades K–2) and **Unlock the Problem** (Grades 3–6)
- **Cross-Curricular Activities**
- **Extend the Math**
- **Literacy and Mathematics**
- **Math in the Real World**
- **Model and Draw**
- **Performance Assessment Tasks**
- **Real-World Videos**

Using and Connecting Mathematical Models and Representations in *GO Math!*

Learners access and communicate mathematics through modeling and representations. ***GO Math!*** © 2015 allows for opportunities to express these ways. Throughout the program, students are able to use manipulatives, organizers, and diagrams. Following are some examples of how this critical strategy is incorporated in ***GO Math!***

Listen and Draw is one feature that encourages modeling in mathematics through listening and then drawing to model or demonstrate what has been learned.

Listen and Draw



Read the following problem aloud. Have children draw a picture to show the problem.

*There are 5 children in the sandbox.
2 walk away. How many children are
in the sandbox now?*

- **What do you draw in the sandbox? Why?**
I draw 5 children because there are 5 children in the sandbox to start with.
- **How many children walk away? 2**
- **How can you show how many children walk away?** Possible answer: I can draw a circle around 2 children and cross them out.

Each lesson provides an opportunity for students to use **GO Math!** dry-erase MathBoards to create visual models to solve problems and record their work. This tool also provides a vehicle for students to explain their thinking and for teachers to observe student work.

Share and Show



The first problem connects to the learning model. Have students use the MathBoard to explain their thinking.

Model and Draw allows students to explore solutions, model, draw, and share.

Model and Draw



MP4 Model with mathematics. Work through the model with children.

- **How many cats are there?** There are 4 cats.
- **How many cats walk away? How do you know?** 1 cat walks away. Possible answer: 1 cat is going the other way; it is moving away from the group.
- **How does the picture show taking from the whole?** Possible answer: The cat walking away is circled and crossed out.

One Way, Another Way, a prompt for teachers, reinforces and encourages students to look for other ways to represent solutions.

One Way

MP6 Attend to precision. Discuss how to line up place values and compare the digits from left to right.

Another Way

Discuss how to compare two numbers using a place-value chart.

Communicating Mathematically in *GO Math!*

Opportunities that encourage talking and writing about mathematics help students to learn, reflect on, and refine mathematical ideas. ***GO Math!*** reinforces the importance of mathematical discourse through communicating mathematically—by talking and writing—and also provides math-specific vocabulary activities for further support. Specific program features that encourage oral and written communication follow.

The **Math Journal** is used to record students' written responses to the math they are learning.

Math Journal *WRITE* Math

Write a word problem that can be solved by drawing an array. Then draw the array and solve the problem.

Math Talk offers a chance for students to talk about math—and an opportunity for teachers to monitor and assess student progress.



MP3 Construct viable arguments and critique the reasoning of others. Use **Math Talk** to focus on children's understanding that there are different ways to take apart numbers.

- How does the number of cubes you color in each row change from top to bottom?

Possible answer: I color one less cube each time.

- How does the number of cubes you cross out change from top to bottom?

Possible answer: I cross out one more cube each time.

Show and Share allows students to show work on their MathBoards and explain it.

Share and Show



The first problem connects to the learning model. Have students use the MathBoard to explain their thinking.

Online iTools provide another way for students to use models and representations to solve problems.



The Write Way provides a chance for students to write about what they know at the start of each chapter.

The Write Way

Reflect

Choose one idea. Write about it.

- Compare and contrast a hundredth and a thousandth. Tell how they are alike and how they are different.
- Explain how to use benchmarks to estimate: $0.28 + 0.71$
- A phone company charges a base fee of \$10 per month. Then, each minute used costs 10 cents more. Use a sequence to tell how much 20, 30, and 40 minutes would cost.
- Write a note to a friend about something you learned in Chapter 3.



In **GO Math!**, students learn the language of mathematics through vocabulary-specific activities. With the **Vocabulary Builder**, students work alone or with partners to complete a vocabulary activity that introduces them to vocabulary essential to learning in the unit.

Vocabulary Builder

Have children complete the activities on the page by working alone or with partners.

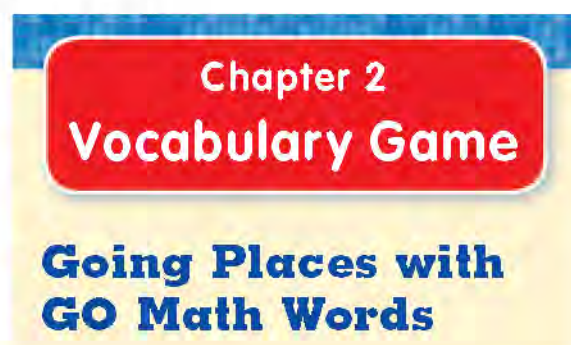
► **Visualize It** Have children sort the review words and record them in the diagram. Have children share how they sorted the words and tell how they decided where to place each one.

► Understand Vocabulary

You may want to share the following concepts with children.

1. When you subtract, you take from a group.
2. When you take 3 apples from 7 apples, there are 4 apples.

Vocabulary Cards are cut-apart cards that students can use to reinforce the language they are learning. As well, each **Vocabulary Game** offers students the chance to have fun with math words. They can play before, during, or after content is taught.



In addition to these examples, there are prompts and suggestions throughout **GO Math!** that reinforce mathematical discourse. Some of these include:

- **Cooperative Grouping on Vocabulary**
- **Developing Math Language**—activities geared to English language learners
- **Literacy and Mathematics**
- **Model Language**—specific suggestions for English language learners

Presenting Purposeful Questions in GO Math!

In **GO Math!** © 2015, teachers are given guidance to ask meaningful questions to students related to the critical concepts they are learning. Within each lesson, teachers are provided with specific, critical questions to ask students, which take form in the following examples.

Beginning each lesson, teachers pose the **Essential Question** and reinforce the question throughout and at the close of the lesson. After beginning, teachers ask additional **Connections** questions—thought-provoking questions to encourage thinking about what students know. Through the **Learning Activity**, teachers explore more about what students know.

Essential Question

How do you solve subtraction problems by making a model?

Making Connections

Invite children to tell you what they know about subtraction.

What are some words from story problems that tell you to subtract?

Possible answers: take away, left

Learning Activity

Direct children to think about ways to model subtraction problems.

- **What is the problem asking you to find?** The number of shovels that are missing.
- **What is the total number of buckets?** 6
- **How many of the buckets have shovels?** 2
- **What word from the story tells you to subtract?** Possible answer: missing

When prompted by **Listen and Draw** at Grades K–2 and **Unlock the Problem** at Grades 3–6, teachers ask more questions that relate to Real-World learning and draw on the Mathematical Practices.

Listen and Draw



Materials connecting cubes

Read the following problem to the class and have children show all the ways to solve it.

Jada has 5 grapes. What are all the ways she can share the grapes with her sister?

Guide children to model, color, and draw all the ways to take apart 5.

- **How can you model subtracting zero?** *I make a 5-cube train to show that no cubes were taken away.*
- **How can you color and draw to show the model?** *I color all of the cubes in the cube train. Since I subtract zero, I do not draw to take any away.*

Discuss with children that this first model shows Jada did not share any grapes with her sister. Next, tell children to circle and cross out to show the grapes Jada gives her sister and color to show the grapes Jada has left.

- **How can you color and draw to show the model?** *I circle and cross out 1 cube. Then I color the rest of the cubes.*
- **How many cubes do you color?** *4 cubes*

Have children continue in a similar manner for the remaining ways.

Building Procedural Fluency from Conceptual Understanding in *GO Math!*

In *GO Math!* © 2015, teachers are given guidance to support both concepts and procedures in mathematics learning. Each lesson and chapter follows an arc of instruction. Students use concrete models to develop meaning of the concepts and then move to creating “quick pictures” or use visual models to develop necessary skills and/or apply to problem solving. Students then practice the skills and concepts developed to build procedural fluency. Practice pages are built into the Student Book for every lesson to help students achieve fluency and confidence with grade-level concepts.

In addition, within lessons of each chapter, *GO Math!* offers **Fluency Builders** to provide suggestions to teachers about how to help students achieve procedural fluency of the concepts they are learning.

Fluency Builder

Common Core Fluency
Standard 3.NBT.A.2

Mental Math Have students practice making sums of 10. Ask questions similar to the following:

- **What number added to 7 equals to 10?**
- **How far away from 10 is 6?**
- **What number do you add to 8 to get 10?**

You may wish to provide number lines so that students can visualize the distances and build their number sense.

In addition to the **Fluency Builder**, there are tasks and activities throughout **GO Math!** that reinforce content and process development, such as: Daily Practice and Homework in the Student Book; Online Teacher Resource book with flashcards and practice sheets; Online Strategies and Practice for Skills and Fact Fluency; and Online Mega Math games. The **Personal Math Trainer**® also provides unlimited algorithmic practice opportunities for students to build fluency.

Supporting Students' Mathematical Struggles in **GO Math!**

In **GO Math!** © 2015, students have numerous opportunities to practice what they are learning, which offers teachers many chances to observe students as they show what they know or struggle with what they do not yet understand.

Unlock the Problem introduces students to real-world situations and applications. Teachers are provided with scaffolding support so they can ask questions that relate to real-world learning and draw on the Mathematical Practices.

Unlock the Problem



MATHEMATICAL PRACTICES

Lead a discussion about why you would use multiplication for this situation.

Help students work through the problem.

- How does the multiplication problem change each time 1 bird leaves each birdbath? Possible answer: the second factor decreases by 1 each time.

MP3 Construct viable arguments and critique the reasoning of others.

- Could you write a multiplication sentence if 1 bird leaves only 1 of the birdbaths? Explain. No; I need equal groups to write a multiplication sentence.
- What multiplication sentence shows that there are no birds in the birdbaths?

$$4 \times 0 = 0$$

There are a variety of opportunities for teachers to observe students as they show what they know—or what they struggle with completing.

Try This! and **On Your Own** are examples of when students can apply what they are learning, and teachers can examine what students know—and guide them through productive struggle.

Try This!

MP5 Use appropriate tools

strategically. In this example, students subtract decimals in two different ways.

A Why is it important to align place values to subtract decimals using paper and pencil?

It ensures that we subtract the correct digits.

B Remind students that they should use an estimate to check the reasonableness of their answer when using a calculator. If their answer is not reasonable, they need to check that they entered the values into the calculator correctly.

On Your Own

If students complete the checked exercises correctly, they may continue with the remaining exercises. Remind students of the importance of checking their work.

MP1 Make sense of problems and persevere in solving them.

- **How can you use an estimate to check the reasonableness of the answer you find for Exercise 7?** Possible answer: Round \$18.39 to \$18, round \$7.56 to \$8, and then compare \$26, the estimate of the sum, to the exact answer.
- **How can you use an inverse operation to check your answer in Exercise 8?** Add 4.39 to the exact answer; if the sum is 8.22, the answer checks.

The **Professional Development Videos** have specific segments in which teachers can observe a classroom showing students engaged in productive struggle and how the teacher can effectively facilitate the problem-solving process. While teachers are aware of student struggles on a regular basis, these videos provide additional support.

Professional Development Videos

- Learn more about the content
- See first-hand the integration of the Math Processes and Practices
- Watch students engaged in a productive struggle

In addition to these examples, there are options for student practice and teacher observation throughout **GO Math!**. Some of these include:

- **Lesson Check**
- **Practice and Homework**
- **Problem of the Day**
- **Spiral Review**
- **Unlock the Problem**

Eliciting and Using Student Work in *GO Math!*

GO Math! © 2015 encourages many points throughout instruction for teachers to collect and assess student work. One particularly helpful section of the program is the **Common Errors** callout where teachers are given guidance about errors students might make and how to help them resolve and learn before those errors become habitual misconceptions.



COMMON ERRORS

Error The number of decimal places in a sum or difference is not correct.

Example Students write a difference of 8.7 when subtracting 7.44 from 8.31.

Springboard to Learning Make sure students understand how estimates and inverse operations can be used to check their answers. An estimate of the difference (1) would have indicated that their answer was not reasonable.

Strand 2: Access and Equity

All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study—and support to learn—mathematics. Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students.

(NCTM Principles & Standards for School Mathematics, 2000, p. 12)

Defining the Strand

Students in today's classrooms come from increasingly diverse backgrounds, in regard to culture and language as well as in their background knowledge, abilities, motivations, interests, and modes of learning (Tomlinson, 2005). Mathematical learning is important to each of these different students; "All young Americans must learn to think mathematically, and they must think mathematically to learn" (National Research Council, 2001, p. 1). To effectively teach mathematics skills and concepts, teachers of mathematics must be knowledgeable of, and sensitive to, the needs of all learners in the mathematics classroom.

In the classroom, teachers encounter students who are on grade, above grade, below grade—as well as English language learners, students with special needs, and students with varying learning styles and cultural backgrounds. As Vygotsky (1978) noted in his seminal research on learning, "Optimal learning takes place within students' 'zones of proximal development'—when teachers assess students' current understanding and teach new concepts, skills, and strategies at an according level." Research continues to support the notion that for learning to take place, activities must be at the right level for the learner (Tomlinson & Allan, 2000; Valencia, 2007). Therefore, teachers must correctly identify each child's needs for instruction and additional support. Differentiation offers teachers a means to provide instruction to a range of students in today's classroom (Hall, Strangman, & Meyer, 2009).

Most importantly, instruction needs to meet the needs of all students. To achieve this universal access, teachers must employ effective assessment practices, diagnose student needs, and assess progress regularly. Teachers must teach intentionally, scaffolding instruction so that all students can access the information.

GO Math! centers on helping all students gain a deeper understanding of mathematics concepts and practices. **GO Math!** supports the achievement of diverse learners by incorporating tactics to meet the varying needs of students through effective differentiation and by providing universal access.

Research that Guided the Development of the GO Math! Program

Differentiated Instruction

As every teacher finds upon entering his or her classroom, students differ in many important ways. As Tomlinson (1997) emphasizes in her discussion of differentiation, "Students are not all alike. They differ in readiness, interest, and learning profile...Shoot-to-the-middle teaching ignores essential learning needs of significant numbers of struggling and advanced learners" (p. 1). In the discussion of how to meet the needs of every learner in the classroom, educators will hear the terms differentiation, universal

access, and universal design. Universal access refers to the idea of providing an equal opportunity for high-quality curriculum and instruction for all students. But how best to achieve this universal access? Differentiation and universal design are terms used to describe two related and complimentary approaches to meeting the needs of all students in the classroom.

In universal design, the needs of all students are considered at the point of instructional design; methods, materials, and assessments (diagnostic and formative) are created to recognize and address the wide range of student needs. In differentiation, modifications take place at the point of instruction; in differentiating instruction, teachers are responsive to what happens in the classroom, and are “flexible in their approach to teaching and adjust the curriculum and presentation of information to learners rather than expecting students to modify themselves for the curriculum” (Hall, Strangman, & Meyer, 2009).

To create an environment in which the barriers that limit comprehensive student access to learning are removed, educators can apply the approaches of universal design. The movement to universal design in education was inspired by the same movement in architecture, to allow access for all (Shaw, 2011). In education, various researchers have employed different terms for related ideas—Universal Design for Learning (UDL), Universal Design for Instruction (UDI), and so on—but one essential characteristic of universal access is that curricular materials be designed to be flexible. Teaching materials should allow for flexible methods of presentation, expression, and engagement by offering multiple examples, employing multiple media and formats, engaging in supported practices, and allowing flexible opportunities for demonstrating skill (Hall, Strangman, & Meyer, 2009; Shaw, 2011). Technology can be particularly beneficial to allow for such flexibility (Hitchcock, Meyer, Rose, & Jackson, 2002).

Differentiating instruction is an organized, but flexible way to alter teaching and learning to help all students maximize their learning (Tomlinson, 1999), and is necessary in order to meet the needs of all learners in today’s diverse classrooms (Tomlinson, 2000). To differentiate instruction, teachers can adjust the content of what is being learned, adjust the process of learning (by providing additional supportive strategies, for example, or adjusting pacing); and tailor the expected outcomes (assessments, products, or tasks) of how learning is assessed (Tomlinson, 2001). In the mathematics classroom, “mathematics instructors must respond to the diverse needs of individual students...using differentiated instruction, a process of proactively modifying instruction based on students’ needs” (Chamberlin & Powers, 2010, p. 113).

Research points to the benefits of differentiation. In a study of numerous teachers using differentiated instruction, researchers found these benefits: students felt learning was more relevant; students were motivated to stay engaged in learning; students experienced greater success; students felt greater ownership of content, products and performances; and teachers gained new insights (Stetson, Stetson, & Anderson, 2007).

Research also shows that differentiation benefits all students—low achievers in mathematics as well as high achievers. In its review of studies on teaching mathematically gifted students, the National Mathematics Advisory Panel (2008) found that:

- The studies reviewed provided some support for the value of differentiating the mathematics curriculum for students with sufficient motivation, especially when acceleration is a component (i.e., the pace and level of instruction are adjusted).

- A small number of studies indicated that individualized instruction, in which pace of learning is increased and often managed via computer instruction, produces gains in learning.

The Panel concluded that gifted students, too, benefit from a differentiated curriculum (National Mathematics Advisory Panel, 2008).

Response to Intervention

Both differentiated instruction and Response to Intervention (RtI) “share a central goal: to modify instruction until it meets the needs of all learners” (Demirsky Allan & Goddard, 2010). According to Demirsky Allan and Goddard (2010), these two instructional approaches are complementary and share the premises that all students have different academic needs and that teachers must teach accordingly to meet these needs—and ensure student success. Like differentiation, “At the heart of the RtI model is personalized instruction, during which each student’s unique needs are evaluated and appropriate instruction is provided, so that students will succeed” (McREL, 2010, p. 15). While differentiation was conceived as a way to respond to the needs of diverse learners in the classroom, RtI was envisioned as a prevention system with multiple layers—a structured way to help students who were struggling before they fell behind their peers—and so it focuses on early, and ongoing, identification of needs and tiers of responses.

RtI is a model that integrates instruction, intervention, and assessment to create a more cohesive program of instruction that can result in higher student achievement (Mellard & Johnson, 2008). RtI is most commonly depicted as a three-tier model where Tier 1 represents general instruction and constitutes primary prevention. Students at this level respond well to the general curriculum and learn reasonably well without additional support. Tier 2 represents a level of intervention for students who are at risk. Students at Tier 2 receive some supplementary support, in the form of instruction or assessment. Tier 3 typically represents students who need more extensive and specialized intervention or special education services (Smith & Johnson, 2011).

According to Griffiths, VanDerHeyden, Parson, and Burns (2006), an effective RtI model should include three elements:

1. Systematic assessment and collection of data to identify students’ needs;
2. The use of effective interventions in response to the data; and
3. Continued assessment of students to determine the effectiveness of interventions—and the need for any additional intervention.

A growing body of research supports the effectiveness of RtI (for example, see Burns, Appleton, & Stehouwer, 2005). Research from Ketterlin-Geller, Chard, and Fien (2008) found that an integrated system, like that of RtI, can lead to improvement in mathematics performance on various achievement measures when used to intervene with students who are underperforming in mathematics. Fuchs, Fuchs, and Hollenbeck (2007) looked at RtI in mathematics with students in Grade 1 (a comprehensive program) and Grade 3 (a focus on word problems). They found that the data supported RtI at both grade levels, and showed “how two tiers of intervention, designed strategically to work in supplementary and coordinated fashion, may operate synergistically to decrease math problem-solving difficulties for children who are otherwise at risk for poor outcomes” (p. 19).

Research suggests that integrating Rtl successfully into classroom instruction involves a number of elements, described in a 2009 publication from the What Works Clearinghouse of the U.S. Department of Education (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009). At the Tier 1 level, all students should be screened to identify those at risk—and interventions for those at risk should be provided. At Tiers 2 and 3, the following are important and proven effective by research:

- The focus of instructional materials in Grades K through 5 should be on whole numbers; in 4 through 8, rational numbers.
- Instruction during intervention should be explicit and systematic, and should include providing models of problem solving, communication about math, practice, feedback, and review.
- Interventions should require students to solve word problems.
- Interventions should provide opportunities for students to work with graphic organizers and visual representations.
- Interventions should devote regular time to arithmetic fact fluency.
- Students' progress should be monitored.
- Interventions should be designed to motivate students (Gersten, Beckmann, Clarke, Foegen, Marsh, Star, & Witzel, 2009).

From Research to Practice

Differentiated Instruction in **GO Math!**

GO Math! supports teachers in implementing effective differentiation so that they meet the varied needs of students in their classrooms. With **GO Math!** practical, point-of-use support is built into each lesson so all learners can achieve success. The program's write-in *Student Edition* allows students to explore concepts, take notes, answer questions, and complete homework right in the textbook, encouraging active learning. Additional videos, activities, and learning aids support students at the point-of-use in the print and online textbook, supporting students' various learning styles.

Multiple program features exist that use varied and flexible multimedia formats to support differentiated instruction throughout **GO Math!** These include:

- The **Grab-and-Go Differentiated Centers Kit** contains countless literature, games, and activities for everyday use in the classroom in tailoring instruction to meet students' needs. The Grab-and-Go!™ Differentiated Centers Kit contains ready-to-use readers, games, and math center activities that are designed for flexible usage. Readers that integrate math skills with cross-curricular content. Games that engage students to practice math skills. Math Center Activities that focus on computation, mental math, geometry, measurement, and challenge activities.
- **Differentiated Instruction** suggestions provided in the **Teacher's Edition** offer ways for teachers to meet the needs of specific populations.
- **Practice** and **Homework** exercises provide ongoing practice for students at different levels of understanding.
- **Reteach** suggestions target ways for teachers to develop the skills of students who may not master concepts the first time around.
- **Enrich, Advanced Learners**, and **Extend the Math** activities in text and online offer challenging extensions for students ready to move ahead.

- **Math on the Spot** video tutorials provide step-by-step instruction of the math concepts covered in each lesson by exploring a higher order problem from the lesson.
- The **Personal Math Trainer** is an online component to practice skills and complete homework, with a variety of learning aids—including videos, guided examples, and step-by-step solutions. Students can explore many ways to improve understanding.
- **Animated Math Models and Mega Math** let students interactively explore and practice key math concepts and skills.
- English language learners are supported specifically with **Strategies for English Learners**, which appear at the beginning of each chapter. Vocabulary activities, games, organizers, and more are included throughout to support students as they develop language and mathematics communication. There are **ELL** callout boxes in each lesson that provide continuous guidance on how to support students.

Response to Intervention in GO Math!

Through print and digital resources, **GO Math!** © 2015 supports a Response to Intervention (RtI) instructional model. Because of the many varied options and resources in the program, teachers can select instructional strategies and resources that specifically align with each student's level of understanding and preferred learning style.

The program offers resources for each level of RtI to diagnose students' intervention levels:

- Tier 1: On-Level Intervention: The **GO Math!** program uses research-based instructional strategies to ensure quality instruction for all students.
- Tier 2: Strategic Intervention: For students who are not responding to on-level instruction, Tier 2 is met with increased time and focus to support struggling learners and reinforce skills that might not have been previously mastered.
- Tier 3: Intensive Intervention: For those students who need more specialized intervention, **GO Math!** offers additional support.

Intervention Options



Response to Intervention

Use Show What You Know, Lesson Quick Check, and Assessments to diagnose students' intervention levels.

TIER 1	TIER 2	TIER 3	ENRICHMENT
On-Level Intervention For students who are generally at grade level but need early intervention with the lesson concepts, use: Reteach (in the <i>Chapter Resources</i>) Personal Math Trainer Tier 1 Activity online	Strategic Intervention For students who need small group instruction to review concepts and skills needed for the chapter, use: Strategic Intervention Guide Personal Math Trainer Prerequisite Skills Activities Tier 2 Activity online	Intensive Intervention For students who need one-on-one instruction to build foundational skills for the chapter, use: Intensive Intervention Guide Personal Math Trainer Prerequisite Skills Activities	Independent Activities For students who successfully complete lessons, use:  Differentiated Centers Kit - Advanced Learners Activity for every lesson - Enrich Activity (in the <i>Chapter Resources</i>) HMH Mega Math

With **GO Math!**, teachers gain valuable insight into student performance through tools and resources at planned points throughout each chapter and lesson.

- **Before the Chapter**—Show What You Know
- **During the Lesson**—Share and Show
- **After the Chapter**—Chapter Review/Test

	Before the Chapter	During the Lesson	After the Chapter
	 Show What You Know • Prerequisite Skills Activities • Personal Math Trainer	 Share and Show • Reteach • Mid-Chapter Checkpoint • Personal Math Trainer • Reteach Activity (online)	 Chapter Review/Test • Reteach • Personal Math Trainer • Reteach Activity (online)

One of *GO Math*'s central resources to help is the **Personal Math Trainer**, which offers **Online Assessment and Intervention**. The program allows teachers to monitor student progress through reports and alerts, and to create and customize assignments aligned to specific lessons or standards. With the **Personal Math Trainer** teachers and students have an online resource for:

- **Practice**—Students get practice on key concepts supported by guided examples, step-by-step solutions, and video tutorials that they can access at any time.
- **Assessments**—Teachers can choose from course assignments or customize their own based on course content, standards, and difficulty levels.
- **Homework**—Students can complete online homework with a wide variety of problem types—and the system can automatically grade the homework to give students and teachers immediate feedback.
- **Intervention**—The **Personal Math Trainer Powered by Knewton™** can automatically prescribe targeted, personalized intervention or enrichment for each student.

Strand 3: Curriculum

A standards-based curriculum combined with the creative use of classroom strategies can provide a learning environment that both honors the mathematical strengths of all learners and nurtures students where they are most challenged.

(McREL, 2010, p. 7)

The standards stress not only procedural skill but also conceptual understanding, to make sure students are learning and absorbing the critical information they need to succeed at higher levels—rather than the current practices by which many students learn enough to get by on the next test, but forget it shortly thereafter...

(National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010b, online)

Defining the Strand

Mathematical learning involves learning content *and* processes. Mathematical *content* relates to the subject of math—what students know and do; mathematical *practices* relate to the vehicles for doing math—how students acquire and use knowledge (NCTM, 2000). According to the National Research Council, linking content and practice—and reflecting both in the mathematics classroom—is essential to student understanding (2001). The Common Core State Standards for Mathematics address content and processes with a balanced approach in which “mathematical understanding and procedural skills are equally important” (NGA & CCSSO, 2010a, p. 4).

Rigor too, is essential, but only if placed on a foundation of strong skills and fluency. While some have suggested that a solution to the problem of low student mathematical skills is to reduce the focus on computation and “simpler” math skills, research suggests that students’ performance on items of low- and high-difficulty correlate highly—suggesting that students’ “mathematical abilities to solve problems at different levels of mathematics rigor are complementary” (Ginsburg, Cooke, Leinwand, Noell, & Pollock, 2005, p. v).

Finally, coherence, or the pattern by which topics are introduced and build across grades, may be “one of the most critical, if not the single most important, defining elements of high-quality standards” (Schmidt et al., 2005, p. 554). The Common Core reflect coherence; designers “drew on research on learning progressions” (Cobb & Jackson, 2011) during development.

The Common Core State Standards for Mathematics accomplish the goals of focus, balance, rigor, and coherence, articulating a rigorous, balanced progression that builds from grade to grade. **GO Math!** © 2015 meets these expectations of the Common Core through a comprehensive mathematics program designed to support teachers in effectively building students’ mathematical knowledge and skills, content and processes.

Research that Guided the Development of the GO Math! Program

Content Standards

The description of standards or instruction as “a mile wide and an inch deep” has become a common way to describe expectations and instruction that cover many topics—but none to mastery. Past comparisons of the U.S. with other countries have suggested that the U.S. K through 8 curriculum is “shallow, undemanding, and diffuse in content coverage” (National Research Council, 2001, p. 4).

In contrast, research suggests that a greater focus on fewer content areas leads to greater mastery. Reviews of the mathematics curriculum in top-performing countries find that they “present fewer topics at each grade level but in greater depth” (National Mathematics Advisory Panel, 2008, p. 20). The Common Core State Standards for Mathematics “promote rigor not simply by including advanced mathematical content, but by requiring a deep understanding of the content at each grade level, and providing sufficient focus to make that possible” (Achieve, 2010, p. 1). Cobb and Jackson (2011) reviewed the standards and came to the conclusion that “the developers make good on their intention to focus on a small number of core mathematical ideas at each grade” (p. 184).

Focusing on specific areas of content does not mean arbitrarily reducing focus on computation and skill building. An analysis of TIMSS and PISA results led researchers to conclude that “the evidence does not support proposals to reduce attention to learning computational and simpler mathematical skills in order to focus on strengthening students’ ability to handle more complicated mathematics reasoning” (Ginsburg, Cooke, Leinwand, Noell, & Pollack, 2005, p. v). Instead, students need to focus each year on developing the skills that will allow them to perform well in low- and high-level problem-solving situations.

International comparisons have shown that American students do not perform as well as students from other countries on assessments of math achievement (see TIMSS study by Gonzales, Williams, Jocelyn, Roey, Katsberg, & Brenwald, 2008, and PISA study by Baldi, Jin, Skemer, Green, & Herget, 2007). In an effort to unpack the specific factors that contribute to this relatively low performance across grade levels, Ginsburg and colleagues concluded that “the distribution of that [instructional] time across mathematics content areas differs in ways consistent with our findings about relative performance across content areas” (Ginsburg et al., 2005, p. v). For example, in comparing time spent on specific content areas, researchers found that “the United States devotes about half the time to its study of geometry—its weakest subject—that other countries spend” (Ginsburg et al., 2005, p. 22). In other words, if teachers want to improve students’ performance across mathematical content areas, they would benefit from focusing instruction accordingly.

Because math learning occurs sequentially, building on previous learning and developing in sophistication, part of a discussion of content in mathematics must address the idea of sequence or progression. As stated previously, the coherence of standards, as illustrated by the logical progression across grade levels, is an essential element of effective standards. Researchers Cobb and Jackson (2011) reviewed the Common Core State Standards for Mathematics and concluded that the standards represent “a major advance in this regard” (p. 184). The standards build on the foundations of earlier years, with new learning extending upon what has already been learned. Strong learning progressions build deep content knowledge and build the complexity of student skills over time.

In the Common Core, the content of the standards in Grades K through 6 builds on students' foundations, preparing them to move on to more demanding math concepts, procedures, and applications.

Critical Areas in the Common Core State Standards for Mathematics, Grades K through 6	
Grade	Critical Areas / Overview
K	1. Representing, relating and operating on whole numbers, initially with sets of objects; and 2. Describing shapes and space.
1	1. Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; 2. Developing understanding of whole number relationships and place value, including grouping in tens and ones; 3. Developing understanding of linear measurement and measuring lengths as iterating length units; and 4. Reasoning about attributes of, and composing and decomposing geometric shapes.
2	1. Extending understanding of base-ten notation; 2. Building fluency with addition and subtraction; 3. Using standard units of measure; and 4. Describing and analyzing shapes.
3	1. Developing understanding of multiplication and division and strategies for multiplication and division within 100; 2. Developing understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; 3. Developing understanding of the structure of rectangular arrays and of area; and 4. Describing and analyzing two-dimensional shapes.
4	1. Developing an understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; 2. Developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and 3. Understand that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.
5	1. Developing fluency with addition and subtraction of fractions and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions) 2. Extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and 3. Developing understanding of volume.
6	1. Connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; 2. Completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; 3. Writing, interpreting and using expressions, and equations; and 4. Developing understanding of statistical thinking.

Mathematical Practices

What is mathematics? By looking at the many interrelated skills and knowledge involved in learning and doing mathematics, it is clear that mathematics is not simply a body of content or topics to be learned.

Developing children's mathematical ways of thinking is an essential element of effective mathematics instruction. "[C]ompetence in a domain requires knowledge of both concepts and procedures. Developing children's procedural knowledge in a domain is an important avenue for improving children's conceptual knowledge in the domain, just as developing conceptual knowledge is essential for generation and selection of procedures" (Rittle-Johnson, Siegler, & Alibali, 2001, p. 359-360). Research by Franke, Kazemi, and Battey (2007) suggests that students need an environment to develop both concepts and skills in order to become flexible when engaging with mathematical ideas, and to develop as critical thinkers.

In attempting to define the many aspects of mathematics learning and understanding, the National Research Council (2001) identified five strands of mathematical proficiency:

- Conceptual understanding*—comprehension of mathematical concepts, operations, and relations;
- Procedural fluency*—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence*—ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning*—capacity for logical thought, reflection, explanation, and justification;
- Productive disposition*—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy. (p. 5).

The group concluded that "The integrated and balanced development of all five strands of mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) should guide the teaching and learning of school mathematics" (National Research Council, 2001, p. 11).

In developing the *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics identified expectations for content as well as for process. Under its Process Standards, NCTM includes Problem Solving, Reasoning and Proof, Communication, Connections, and Representation.

The Common Core State Standards for Mathematics are an extension of these earlier efforts, by NCTM and the NRC, to define the processes and proficiencies of mathematics. In the Common Core State Standards for Mathematics, the Standards for Mathematical Practice, "describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years" (NGA & CCSSO, 2010a). Students meet the Standards for Mathematical Practice by demonstrating the ability to:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.

- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

From Research to Practice

Content Standards in *GO Math!*

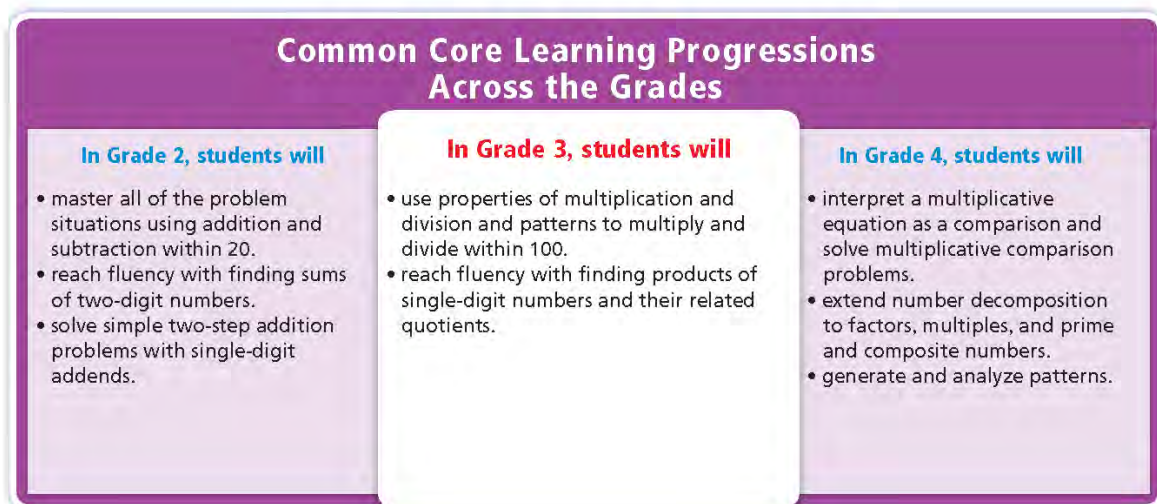
The Common Core State Standards reflect focus, coherence, and rigor, which is reflected in Houghton Mifflin Harcourt's *GO Math!* © 2015. Throughout *GO Math!* © 2015, a strong focus is maintained on the content and practices of the Common Core State Standards for Mathematics.

Common Core assessment consortia have designated standards clusters as major, supporting, or additional. Clusters designated as major should compose the major work of each grade. The lessons in *GO Math!* reflect the progressions and emphasis on the major work with students spending the majority of their time on the major work at their grade level. Supporting and Additional Clusters serve to support this focus on the major work.

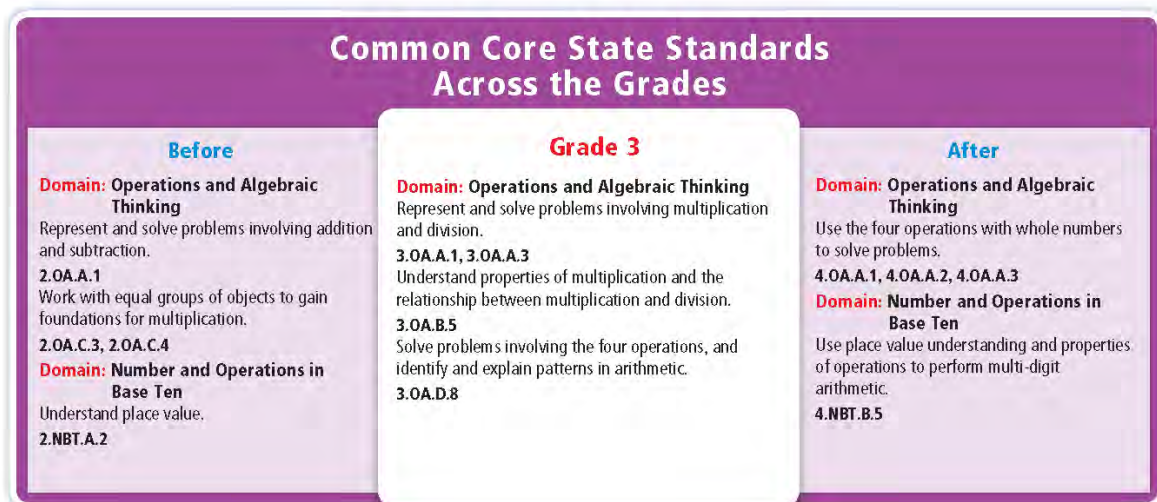
For each grade level, correlations for the *GO Math!* program with the **Common Core State Standards for Mathematics** are provided in the Planning Guide, the Table of Contents and within every lesson.

Critical Area		Operations and Algebraic Thinking	
		Common Core CRITICAL AREA Developing understanding of addition, subtraction, and strategies for addition and subtraction within 20	
Chapter 1	Addition Concepts	9	
Chapter 2	Subtraction Concepts	65	
Lessons	Grade 1 Common Core State Standards		
	Domain: Operations and Algebraic Thinking	1.OA	
2.1–2.4, 2.6, 2.8	Cluster A: 1.OA.A.1	Represent and solve problems involving addition and subtraction. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	
2.9	Cluster C: 1.OA.C.6	Add and subtract within 20. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., $8 + 6 = 8 + 2 + 4 = 10 + 4 = 14$); decomposing a number leading to a ten (e.g., $13 - 4 = 13 - 3 - 1 = 10 - 1 = 9$); using the relationship between addition and subtraction (e.g., knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$); and creating equivalent but easier or known sums (e.g., adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$).	
2.5, 2.7	Cluster D: 1.OA.D.8	Work with addition and subtraction equations. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.	

In **GO Math!**, strong **Common Core Learning Progressions** help students develop a deep understanding of mathematical content. Through the program, students build to more complex skills and content knowledge over time.



In **GO Math!** © 2015, all learning is shown in the context of a progression of skills—prerequisite skills, knowledge, and vocabulary is made explicit and students and teachers can clearly see the expected progression of learning **Across the Grades**. In the Teacher Edition, chapters include graphic presentations of prerequisites and expected outcomes for learning.



Mathematical Practices in *GO Math!*

The ***GO Math!*** © 2015 program provides balanced instruction on mathematical content and practices. In ***GO Math!***, instructional time is devoted to developing both students' content skills as well as their mathematical practices. Numerous program features build students' mathematical practices, including:

- **Analyze Relationships**
- **Check for Reasonableness**
- **Communicate Mathematical Ideas**
- **Common Errors**
- **Go Deeper**
- **Listen and Draw**
- **Math Journal**
- **Math Talk**
- **Model and Draw**
- **Problem Solving Applications**
- **Real-World Problems**
- **Reflect**
- **Share and Show**
- **Teaching for Depth**
- **Think Smarter**
- **Try Another Problem**
- **Unlock the Problem**
- **Use Tools**

For specific examples of how the ***GO Math!*** program supports the Standards for Mathematical Practice, see the examples that follow.

Within the Teacher Edition for each grade level, the **Standards for Mathematical Practices** are first seen in the Table of Contents, so teachers are aware that the practices are embedded in the chapters and lessons.

Table of Contents	
Place Value and Operations with Whole Numbers	Page
Big Idea At A Glance	1A
Digital Resources	1C
Real World Project	1
Chapter 1 Place Value, Addition, and Subtraction to One Million	
Math Processes and Practices	
MPP1 Problem Solving	MPP5 Use Mathematical Tools
MPP2 Abstract and Quantitative Reasoning	MPP6 Use Precise Mathematical Language
MPP3 Use and Evaluate Logical Reasoning	MPP7 See Structure
MPP4 Mathematical Modeling	MPP8 Generalize

Prior to each lesson, teachers are reminded of the **Mathematical Practices** that can be applied to improve student understanding.

FOCUS
COHERENCE
RIGOR

LESSON AT A GLANCE

FCR Focus:
 **Common Core State Standards**
5.NBT.A.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
MATHEMATICAL PRACTICES
MP4 Model with mathematics. **MP5** Use appropriate tools strategically.
MP7 Look for and make use of structure.

Learning Objective
Model, read, and write decimals to thousandths.

FCR Coherence:
Standards Across the Grades
Before Grade 5 After
4.NBT.A.1 5.NBT.A.1 6.NS.B.3
4.NBT.A.2
4.NBT.A.3

Language Objective
Student partners describe the relationship between two decimal place-value positions to another pair.

FCR Rigor:
Level 1: Understand Concepts.....*Share and Show*  **Checked Items**
Level 2: Procedural Skills and Fluency.....*On Your Own*
Level 3: Applications.....*Think Smarter and Go Deeper*

Materials
MathBoard, color pencils, straightedge

FCR For more about how *GO Math!* fosters **Coherence** within the Content Standards and Mathematical Progressions for this chapter, see page 149J

Embedded within each lesson is a prompt that provides teachers with suggestions on how they can incorporate the **Mathematical Practices** as well.

Problem Solving • Applications

MP8 Look for and express regularity in repeated reasoning. Tell students to think about how the value of each place-value position changes as you move to the left or right in a number. To find a number 10 times as much, in which direction would they shift the digits?

Strand 4: Tools & Technology

Research on instructional software has generally shown positive effects on students' achievement in mathematics as compared with instruction that does not incorporate such technologies.

(National Mathematics Advisory Panel, 2008, p. 50)

Defining the Strand

At its most basic, technology can refer to any tools, inventions, or techniques that help us solve problems or perform activities. Technology has always played a role in mathematical learning and study, and can serve as a valuable tool in teaching and learning. Technology can support students' development of skills, exploration and communication of concepts, and ability to reason and problem-solve. With advances in technology, specifically in graphics technology and information technology, new opportunities for mathematical teaching and learning are constantly emerging.

According to the findings of the National Research Council's 2001 review, "research has shown that instruction that makes productive use of computer and calculator technology has beneficial effects on understanding and learning algebraic representation" (p. 420). Several studies and meta-analyses support the use of computers in the classroom to improve student learning (see Britt & Aglinskias, 2002; Li & Ma, 2010; Means, Toyama, Murphy, Bakia, & Jones, 2009; North Central Regional Educational Laboratory, 2003; Teh & Fraser, 1995). Studies point to the effectiveness of computer-based instruction in mathematics particularly. In their meta-analysis, Cheung and Slavin (2011) found that educational technology applications had a positive effect on student assessment performance in mathematics. Sosa, Berger, Saw, and Mary (2011) found that computer-assisted instruction yielded larger student learning effects in statistics. Incorporating computer technology into instructionally rich classrooms—those that value complex thinking, concept application, and problem solving—increases students learning (Hyun & Davis, 2005; Bessier, 2006) in the primary grades (Hyun & Davis, 2005) and in the upper elementary grades (Tiene & Luft, 2002).

Today's "digital natives" (Prensky, 2001) use technology daily (Rideout, Foehr, & Roberts, Kaiser Family Foundation, 2010), and while 94% of students believe technology will improve their school and workplace opportunities, only 39% believe that their school meets their expectations for technology (CDW, 2011).

Clearly, technology is an effective tool to reach today's students. **GO Math!** was developed to take advantage of the instructional benefits of technology and engage and support student learning through multimedia and varied digital tools. Specific program resources are described on the following pages.

Research that Guided the Development of the GO Math! Program

Multimedia Learning

In Mayer's second edition (2009) of *Multimedia Learning*, he again lays out the case for multimedia learning, presenting a cognitive theory of multimedia learning and citing the results of numerous, systematically designed studies which demonstrate the ways in which people learn more deeply from words and visuals rather than from verbal messages alone. According to Mayer, "the case for multimedia

learning is based on the idea that instructional messages should be designed in light of how the human brain works" (Mayer, 2001, p. 4). Mayer (2001, 2005), a leading researcher in the field of multimedia learning, argues that student learning is increased in multimedia environments because information can be presented in multiple formats—including words, audio, and pictures. Students are able to learn and retain information more effectively when they can access information using these different pathways.

Studies in mathematics suggest that digital instruction has specific benefits to the mathematics learner. Meta-analyses looking at the benefits of digital instruction in the mathematics classroom have found positive effects on learning and achievement (Cheung & Slavin, 2011; Li & Ma, 2010). In investigating the efficacy of a computer program to teach geometrical concepts of reflection and rotation, Dixon (1997) found that students in the "dynamic instructional environment outperformed students experiencing a traditional instructional environment on content measures involving reflections and rotations" (p. 356). Taconis (2013) concluded that computers are an excellent tool for teaching problem solving, and are effective for "delivering a variety of learning tasks, worked problems, and exercise problems that focus on strengthening the knowledge base and thinking skills..." (p. 381). Weiss, Kramarski, and Talis (2006) examined the impact of multimedia activities on the mathematics learning of young children and found that students who engaged in multimedia learning either individually or in cooperative learning groups significantly outperformed control group students. In a study which compared users of classroom computer games with a control group, Kebritchi, Hirumi, and Bai (2010) found that the games had a "significant positive effect on students' mathematics achievement" (p. 435).

Multimedia learning opportunities can help to close achievement gaps between groups of students and can be particularly effective with average and with lower-achieving students (see Huppert, Lomask, & Lazarowitz, 2002; Mayer, 2001; White & Frederiksen, 1998). Means, Toyama, Murphy, Bakia, and Jones (2009) found that online learning approaches were effective across types of learners. Computer technologies offer powerful tools for teachers seeking to differentiate and provide for universal access to learning in the classroom; "With the power of digital technologies, it is possible to provide a malleable curriculum in which content and activities can be presented in multiple ways and transformed to suit different learners" (Hitchcock, Meyer, Rose, & Jackson, 2002, p. 9). Multimedia learning environments are able to reach students who learn in different ways—visual learners, auditory learners, kinesthetic learners.

From Research to Practice

Multimedia Learning in *GO Math!*

GO Math! employs technology to support instruction and enhance student learning.

For Students—**GO Math!** offers the **Interactive Student Edition**, which is an interactive, multi-sensory math experience with videos, activities, tools, direct links, and learning aids.



Interactive Student Edition



- Immerses students in an interactive, multi-sensory math environment
- Enhances learning with scaffolded, interactive instruction and just-in-time feedback
- Provides audio reinforcement for each lesson
- Makes learning a two-way experience, using a variety of interactive tools

Math on the Spot video tutorials provide students with step-by-step instruction of the math concepts covered in each lesson by exploring a higher order problem from the lesson.



Math on the Spot Videos

- Models good problem-solving thinking in every lesson
- Engages students through interesting animations and fun characters
- Builds student problem-solving proficiency and confidence
- Builds the skills needed for success on high-stakes assessments

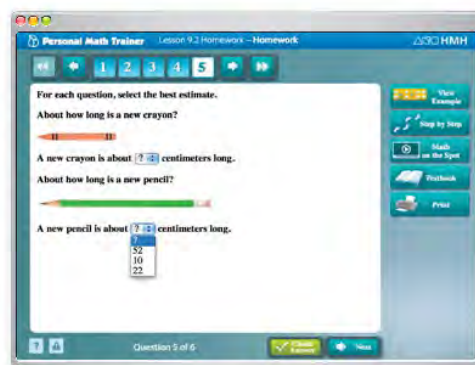


The **Personal Math Trainer** allows students to practice skills, complete homework, and take assessments online. In addition, the **Personal Math Trainer** provides a variety of learning aids that develop and improve students' understanding of math concepts including videos, guided examples, and step-by-step solutions.



Personal Math Trainer

- Creates a personalized learning path for each student
- Provides opportunities for practice, homework, and assessment
- Includes worked-out examples and helpful video support
- Offers targeted intervention and extra support to build proficiency and understanding



For Teachers—**GO Math!** digital features include the **Teacher Digital Management Center**, in which teachers have full online access to lessons, videos, activities, and more at any time. In addition, teachers have access to **Professional Development Videos**, which are on-demand videos related to the Common Core State Standards, Performance Tasks, and Mathematical Practices, including first-hand integration of the practices and students engaging in productive struggles.

Digital Management System

- Manage online all program content and components
- Search for and select resources
- Identify resources based on student ability and needs
- View and assign student lessons, practice, assessments, and more



Strand 5: Assessment

Assessment...refers to all those activities undertaken by teachers—and by their students in assessing themselves—that provide information to be used as feedback to modify teaching and learning activities...

(Black & Wiliam, 1998a, p. 140)

Defining the Strand

Assessment is an essential part of instruction and a process by which teachers can continuously gauge student understanding. Teachers can collect a variety of evidence from students before, during, and after instruction to best meet students' needs. While timing is one aspect of assessment, variety in item types and tasks is also critical to a comprehensive picture of student understanding.

Effective assessment tools allow teachers to collect data about what is working—and what is not—so that they can take precise, swift, and effective action in meeting the specific needs of students. Formative assessment has a positive effect on learning (Black & Wiliam, 1998b; Cotton, 1995; Jerald, 2001). As noted by numerous research studies, the regular use of assessment to monitor student progress can mitigate and prevent mathematical weaknesses and improve student learning (Clarke & Shinn, 2004; Fuchs, 2004; Lembke & Foegen, 2005; Skiba, Magnusson, Marston, & Erickson, 1986). In their research, Baker, Gersten, and Lee (2002) concluded that “providing teachers and students with information on how each student is performing seems to enhance...achievement consistently” (p. 67). There is agreement that “assessment should be more than merely a test at the end of instruction to see how students perform under special conditions; rather, it should be an integral part of instruction that informs and guides teachers as they make instructional decisions” (National Council of Teachers of Mathematics, 2000, p. 1).

Research also points to the importance of using varied item types and tasks in order to get the best reflection of student understanding. As noted by McREL (2010) “Using multiple types of assessments provides more insight into students' learning because students have more than one way to demonstrate their knowledge and skills” (p. 44).

GO Math! supports instruction based on student performance. Throughout the program, varied assessments provide valuable information about student learning that can help teachers plan and modify instruction. Specific examples of how **GO Math!** integrates effective assessment practices are provided on the following pages.

Research that Guided the Development of the GO Math! Program

Diagnostic Assessment

Effective instruction depends upon teachers who make good decisions about how best to meet their students' needs. To make these kinds of decisions, teachers need information that they can trust about students' strengths and weaknesses, knowledge, and understandings. In an instructional context, a *diagnostic assessment* is one in which “assessment results provide information about students' mastery of relevant prior knowledge and skills within the domain as well as preconceptions or misconceptions about the material” (Ketterlin-Geller & Yovanoff, 2009, p. 1). A screening tool given to every student in a

given grade at the opening of the school year can help to identify those who are at-risk or need additional support (Fuchs & Fuchs, 2006).

Studies attest to the benefits of using valid diagnostic measures—and tailoring instruction and supplemental practice according to the results of the diagnostics (for example, see Mayes, Chase, & Walker, 2008). Today's classrooms have disparity in students' prerequisite skills and knowledge and preparation and diagnostic assessment can help to identify the best instructional approach for each student at the outset so that instructional time is not wasted.

Formative Assessment

"Effective instruction depends on sound instructional decision-making, which in turn, depends on reliable data regarding students' strengths, weaknesses, and progress in learning content..." (National Institute for Literacy, 2007, p. 27) The phrase *formative assessment* encompasses the wide variety of activities—formal and informal—that teachers employ throughout the learning process to gather this kind of instructional data to assess student understanding and make and adapt instructional decisions. Its purpose is not an end in itself—such as the assignment of a grade—but rather, the purpose is to guide instruction. Formative assessment moves testing from the end into the middle of instruction, to guide teaching and learning as it occurs (Shepard, 2000; Heritage, 2007). Formative assessment shifts the way that students view assessments—"Assessment should not merely be done to students; rather, it should also be done for students, to guide and enhance their learning" (NCTM, 2000, p. 22).

Educators agree on the benefits of ongoing assessment in the classroom. "Well-designed assessment can have tremendous impact on students' learning ... if conducted regularly and used by teachers to alter and improve instruction" (National Research Council, 2007, p. 344). In its review of high-quality studies on formative assessment, the National Mathematics Advisory Panel (2008) found that "use of formative assessments benefited students at all ability levels" (p. 46). Several reviews of instructional practices used by effective teachers have revealed that effective teachers use formal (such as quizzes or homework assignments) and informal tools (such as discussion and observation) to regularly monitor student learning and check student progress (Cotton, 1995; Christenson, Ysseldyke, & Thurlow, 1989). A meta-analytic study by Baker, Gersten, and Lee (2002) found that achievement increased as a result of regular assessment use: "One consistent finding is that providing teachers and students with specific information on how each student is performing seems to enhance mathematics achievement consistently...The effect of such practice is substantial" (p. 67). In a study of student learning in a multimedia environment, Johnson and Mayer (2009) found that students who took a practice test after studying multimedia material outperformed students who studied the material again (without the assessment).

An additional benefit of formative assessment is that it has been shown to be particularly helpful to lower-performing students. Gersten and Clarke (2007) conveyed similar findings for lower-achieving math students, concluding that "the use of ongoing formative assessment data invariably improved mathematics achievement of students with mathematics disability" (p. 2). In this way, use of formative assessments minimizes achievement gaps while raising overall achievement (Black & Wiliam, 1998b).

Summative Assessment

While traditionally summative assessment is associated with high stakes testing, there is a role for summative assessments in the classroom—as they act as an additional constructive measure. Understanding student learning at the end of a unit or a chapter offers insight when used as a point of information for subsequent instruction as noted by Carnegie Mellon’s Eberly Center Teaching Excellence and Educational Innovation (online).

“Tests given in class...are also important means of promoting feedback. A good test can be an occasion for learning” (Black & Wiliam, 1998, p. 8). When summative results are elicited and interpreted—and teachers take action on that interpretation—that “action will then (directly or indirectly) generate further evidence leading to subsequent interpretation and action, and so on (Wiliam, 2000, Presentation). Teachers can use summative assessments as another measure, another point in time, and another means by which to best evaluate student understanding.

Performance-Based Assessment

Using performance-based assessments or problem solving tasks in the classroom is another effective way to assess student understanding—and encourage critical thinking. Research indicates that high-quality tasks foster students’ abilities to reason, solve problems, and conjecture (Matsumura, Slater, Peterson, Boston, Steele et al., 2006). Students can gain a deeper understanding of mathematics by exploring and reasoning through performance-based tasks.

Items in which students were asked to construct a response—rather than choose among options for answer choices—were shown to involve greater cognitive effort in a study by O’Neil and Brown (1998).

Varied Assessment Types and Options

One single assessment or type of assessment cannot serve all of the purposes of assessment. Research supports that looking at multiple means of assessment is the best way to capture a whole picture of student learning. As noted by Krebs’ (2005) research, using one data point, such as written responses, to evaluate and assess students’ learning can be “incomplete and incorrect conclusions might be drawn...” (p. 411). In addition, “using multiple types of assessments provides more insight into students’ learning because students have more than one way to demonstrate their knowledge and skills” (McREL, 2010, p. 44). Therefore, variety in assessment item types is an integral part of an effective mathematics program.

Asking students to respond to open-ended questions—in writing or through classroom discussion—is another useful way to assess what students are learning. As discussed by Moskal (2000) in her guidelines for teachers for analyzing student responses, students’ responses to open-ended questions afford them the opportunity to show their approaches in solving problems and expressing mathematically what they know, which in turn allows the teacher to see the students’ mathematical knowledge. Research by Aspinwall and Aspinwall (2003) on using open-writing prompts supports the use of open-ended questions in assessment in the mathematics classroom: “Students’ responses to open-ended questions offer opportunities for understanding how students view mathematical topics...this type of writing allows teachers to explore the nature of students’ understanding and to use this information in planning instruction” (p. 352-353). Similarly, by asking students to respond to open-ended questions verbally,

researchers Gersten and Chard (2001) found that “encouraging students to verbalize their current understandings and providing feedback to the student increases learning.”

Multiple-choice items can play an important role in an assessment system as well. The National Mathematics Advisory Panel (2008) found that formative assessments based on items sampled from important state standards objectives resulted in “consistently positive and significant” effects on student achievement (p. 47). In addition, the Panel found multiple-choice items to be equally valuable in assessing students’ knowledge of mathematics (National Mathematics Advisory Panel, 2008).

From Research to Practice

Diagnostic Assessment in *GO Math!*

Specific features and tools in *GO Math!* support teachers in using diagnostic assessment effectively to assess students’ need for instruction. The program’s assessment system includes these diagnostic measures:

- **Prerequisite Skills Inventory**—available in the *Chapter Resources*, these assessment items should be given at the start of the year or when a new student arrives. Test results help to gauge what students know as related to the standards for each grade level.
- **Beginning-of-Year Test**—also available in the *Chapter Resources*, these assessment items should be administered at the start of the year to determine which on-grade skills students already understand.
- **Show What You Know**—available in the *Student Edition*, this section helps determine students’ prerequisite skills and prior knowledge for each chapter.

Formative Assessment in *GO Math!*

GO Math! assessments identify students’ strengths and their weaknesses so that teachers can focus instruction accordingly. *GO Math!* includes a multitude of formative measures including:

- **Quick Check**—available in every lesson of the Teacher Edition, these checks help to monitor student understanding of the content being presented.
- **Mid-Chapter Checkpoint**—also available in the Student Edition, these checkpoints provide opportunities to monitor students’ progress and allow for adjustments for instruction as needed to help students meet content goals.
- **Middle-of-the-Year Test**—available in the *Chapter Resources*, items assess the same standards as the **Beginning-of-Year Test** to provide further insight on student progress toward the content goals.
- **Portfolio**—encourages a collection of student work to reinforce progress and achievements.

Summative Assessment in *GO Math!*

Summative Assessments are incorporated in the ***GO Math!*** program. Summative assessments are another indicator of what students know and can do in mathematics so that teachers can examine performance and make decisions to best meet student needs. Summative measures in ***GO Math!*** include:

- **Chapter Review/Tests**—available in the Student Edition, these measures indicate whether or not additional instruction and practice is necessary for students to master the content taught in each chapter. The test items are presented in a variety of Common Core assessment formats.
- **Chapter Tests**—found in the *Chapter Resources*, these tests evaluate the Common Core standards taught in each chapter. Again, items reflect those of Common Core assessment formats.
- **Performance Assessment Tasks**—also available in the *Chapter Resources*, these tasks contain several opportunities for students to demonstrate and display what they have learned. Available for each chapter and critical area, they are accompanied by teacher supports, scoring rubrics, and examples of student work.
- **End-of-the-Year Test**—available in the *Chapter Resources*, items assess the same standards as the **Beginning-of-Year** and **Middle-of-Year Tests** to provide further insight on student progress toward the content goals. They serve as the final benchmark test for the grade level.

Online Assessment in *GO Math!*

Personal Math Trainer can be used to give the diagnostic, formative, and summative assessments referenced above in an online setting.

Performance Assessment in *GO Math!*

As noted previously, performance assessments are included in the ***GO Math!*** program to offer yet another measure by which to determine student understanding in learning. Performance assessments and tasks allow students to demonstrate their knowledge by applying what they know to a particular task or situation. **Performance Assessment Tasks** are aligned with each Critical Area and each chapter. Teachers can use them to continue to document student growth.

Varied Assessment Types and Options in *GO Math!*

Throughout the ***GO Math!*** program, multiple effective types of assessment appear in an effort to best allow students to demonstrate their knowledge and skills. ***GO Math!*** features strong performance task assessments, as well as multiple-choice items, constructed-response tasks, and item types that mirror those on Common Core assessments.

In addition to the robust assessment resources noted previously, ***GO Math!*** also includes **Test Preparation** books. These books have item formats that represent those that students will encounter on Common Core assessments—specifically Partnership for Assessment of Readiness for College and Career (PARCC™*) and Smarter Balanced Assessment Consortium (SBAC*).

**This product is not endorsed by nor affiliated with PARCC or Smarter Balanced Assessment Consortium.*

Strand 6: Professionalism

Professional teachers are reflective and constantly evaluating their choices and actions to improve instruction...those who understand current trends in education, actively seek opportunities to grow professionally, participate in meaningful professional development activities, initiate changes if appropriate/necessary, and are lifelong learners.

(Definitions from Teachers, from Tichenor & Tichenor, 2005, p. 89)

Defining the Strand

In his book *Good to Great*, Collins (2001) succinctly describes a professional as an individual who is committed to improvement—one who does not accept the status quo, even if it is reasonably good, and chooses to continue to learn and grow.

Other researchers have pointed out that being a professional teacher is a process that develops over time (Clement, 2002; Seifert, 1999). Wise (1989) suggests that professional teachers are those [who] have a firm grasp of the subjects they teach and are true to the intellectual demands of their disciplines. They are able to analyze the needs of the students for whom they are responsible. They know the standards of practice of their profession. They know that they are accountable for meeting the needs of their students (p. 304–305).

Tichenor and Tichenor (2004) conducted a study in which they worked with numerous teachers to gauge how they define “professionalism.” While many definitions were similar to those quoted above, one thread was consistent—“teachers have high standards, ideals, and expectations for themselves and other teachers” (p. 94).

GO Math! supports teacher professionalism. Throughout the program, valuable resources are available to help teachers grow professionally to plan, deliver, and modify effective instruction to maximize student learning. Specific examples of how **GO Math!** promotes teacher professionalism are provided on the following pages.

Research that Guided the Development of the GO Math! Program

Several studies have shown that supporting teachers’ professional growth leads to improvements in student performance (Avalos, 2011; Buczynski & Hansen 2010; Meiers & Ingvarson, 2005; Desimone et al., 2002; Garet et al., 2001, and others).

A study involving hundreds of teachers and thousands of students revealed that professional development had a positive effect on student performance and teacher learning (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). When prompted, teachers were able to generate more effective strategies to help their students than their counterparts. Taking time to examine, reflect on, and learn about instruction leads to greater knowledge for teachers and greater benefits for students.

Early research has shown that online, computer-based professional development can have a positive impact on teaching and learning (Bahr et al., 2004; Benson et al., 2004; Magidin et al., 2012; Rienties, Brouwer, & Lygo-Baker, 2013; Cho & Rathbun, 2013). Providing teachers with professional learning

opportunities that are readily and easily available allows for flexibility in time and need. Teachers can access the resources they need based on their own schedules, and they can determine which resources best suit their needs.

A study by Shaha and Ellsworth (2013) specific to the effects of on-demand, online professional learning found that “schools with higher teacher engagement (e.g. quantity and quality of utilization, participation) significantly outperformed their lower engagement counterparts in 1) student achievement and 2) measures of school and educator success (e.g. teacher retention, student discipline)...conclusions were that higher levels of utilization, engagement, and active use are correlated with higher student achievement and successes for educators and schools” (p. 19).

One specific type of online, on-demand resources that has shown benefits is the use of videos or clips of professional learning. Viewing video clips allows teachers the opportunity to reflect on instructional practices and content (Marzano et al., 2012). As noted by Hiebert et al. (2003), using video clips or recordings can also reinforce the message that teaching mathematics is not just an isolated practice—reflecting on instruction is an opportunity to improve professionally.

From Research to Practice

Professionalism in *GO Math!*

Throughout the *GO Math!* © 2015 program, teachers are provided with numerous opportunities and options for furthering their professional learning. From information on mathematics content to the mathematical practices, resources are readily available to enhance student learning. Examples of sections devoted to teacher development include the following.

At the start of each chapter, written by experts in the field of mathematics, **Teaching for Depth**, provides additional insight into the content.

Chapter 3

Understand Multiplication



PROFESSIONAL DEVELOPMENT

Teaching for Depth

by Juli K. Dixon
Professor of Mathematics Education
University of Central Florida
Orlando, Florida

Meaning of the Factors
Understanding what the factors mean is critical to building an understanding of multiplication.

- In addition and subtraction problems, the addends represent the same type of quantity. That is, if we add or subtract marbles, both addends represent marbles as does the sum.
- With multiplication, if we want to know how many total marbles we have in 5 bags of marbles with 6 marbles in each bag, one factor represents the number of marbles in each bag and the other factor represents the number of bags. The product represents the total number of marbles in all the bags.



Use Models to Explore Properties
Models are useful in making sense of multiplication.

- Modeling multiplication by using groups of objects illustrates how each factor has a specific meaning in multiplication.
- The array model for multiplication leads students to understand the Commutative Property of Multiplication. For example, 3 rows of 4 objects results in the same number of objects as 4 rows of 3 objects. This helps students derive new facts from facts that they already know.



From Models to Efficient Strategies
Students' early explorations of multiplication are similar to their initial experiences with addition and subtraction.

- Students begin by modeling the action of the problem.
- Eventually, students move from direct modeling strategies to more efficient strategies based on counting, addition and subtraction, and derivations of products they know (Kilpatrick, Swafford, & Findell, 2001; Carpenter et al., 1999).

Mathematical Practices
Students make sense of multiplication as finding the total number of objects in all. They use drawings to combine equal groups, skip count on a number line, draw diagrams, and use arrays. As students solve multiplication problems presented in context, they make sense of the meanings of factors and products. Students **look for and express regularity in repeated reasoning** as they apply the Commutative Property of Multiplication to find products by using other known products.

From the Research
“Modeling multiplication problems with pictures, diagrams, or concrete materials helps students learn what the factors and their products represent in various contexts.”
(NCTM, 2009, p. 151)

Professional Development Videos
Multiplication and Division: Strategies and Facts, Grades 3–6, Segments 1, 2, and 5


137E Chapter 3

Within each lesson of each chapter, **About the Math** shares ideas and strategies for teaching mathematics content and practices.

About the Math
Professional Development

Why Teach This
Understanding equal groups and how they can be represented builds the foundation for understanding the operation of multiplication. In a multiplication expression, the first factor is the number of equal groups and the second factor is the number of objects in each group.
The focus of this lesson is on modeling and drawing equal groups to find how many in all. Students do not record multiplication expressions or sentences yet. Equal groups are presented in context, and students use the models to represent and solve word problems.
The goal is for students to recognize and represent situations with equal groups so in later lessons, they will know when multiplication can be used to solve a problem.

 Professional Development Videos

In addition, **Professional Development Videos** are available so that teachers have online, on-demand access to information to help plan for instruction. Each video shares insight on mathematics teaching, and teachers have the flexibility to view, reflect, and plan for instruction when it works best for them.



The **GO Math!** © 2015 program supports teachers as they continue to develop as professionals. In addition to those features noted previously, each chapter and each lesson contains additional suggestions to help teachers balance instruction and build on what they know, including:

- **Go Deeper**
- **Math Talk in Action**
- **Mathematics Practices in Your Classroom.**
- **Think Smarter**

GO Math! © 2015 empowers all educators by offering professional development resources to meet a variety of needs for educators with busy schedules. This includes Initial Implementation Workshops; Advanced Implementation Workshops; eLearning Professional Development and Customized Professional Development.

Case Studies

Brentwood Union Free School District

School District Overview

Brentwood Union Free School District is a suburban district in Brentwood, New York. The district includes 11 elementary schools and 6 secondary schools with a current enrollment of over 17,000 students in Grades K–12. The professional staff includes 1,000 teachers, with approximately 1,100 support personnel and administrators.

Seventy-nine percent of the students in Brentwood Union Free School District are Hispanic, and the remaining student body consists of students identified as Black (12%), White (6%), Asian (2%), or other (1%). Eighty-five percent of the students qualify for free/reduced lunch and 13% percent of the students have an Individualized Education Program (IEP).

New York State Testing Program

The annual New York State Test is taken by all students in Grades 3–8, in the spring. The test includes assessment of ELA, math, and science. The test focuses on how well students have acquired the skills and knowledge described in the state Common Core Learning Standards.

At Grades 3–5, the math New York State Test is comprised of multiple choice, short-constructed response, and extended response items covering five domains equally distributed across the tests: Geometry, Measurement and Data, Numbers and Operations in Base Ten, Number and Operations-Fractions, and Operations and Algebraic Thinking.

The New York State Test scores are categorized into one of four levels of mastery including Level 1 (Well below Proficient), Level 2 (Below Proficient), Level 3 (Proficient), and Level 4 (Above Proficient). Students scoring at Levels 3 and 4 are said to excel and demonstrate knowledge, skills, and practices embodied by the New York State P–12 Common Core Learning Standards for Mathematics.

Student Success with **GO Math!** © 2012

To determine if HMH® **GO Math!**® had an impact on learning, NY State mathematics test scores from the Spring of 2013, prior to using the program and the Spring of 2014, after one year of usage, were obtained for Grades 3, 4, 5, and 6. These findings are presented in Figure 1.

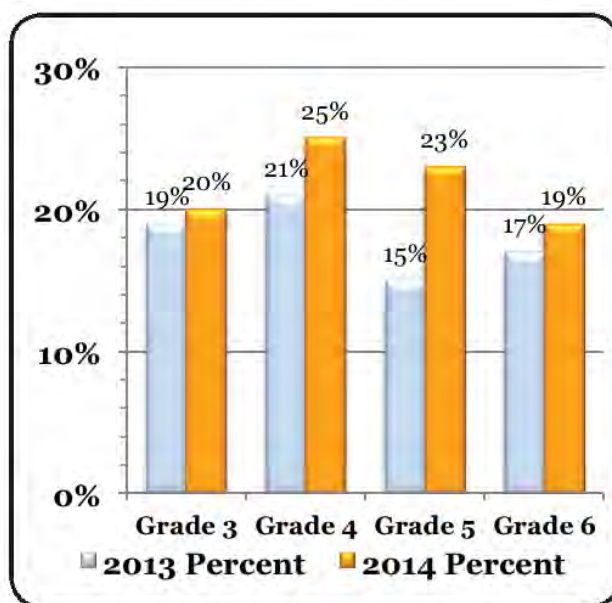


Figure 1

Percent of Students Level 3/Level 4

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Level 3 or higher was greater in 2014, after using **GO Math!**, than the previous year with an average gain of over 3%.

Conclusion

This one-year examination of the implementation of **GO Math!** © 2012 at Brentwood Union Free School District revealed that the program was associated with increased mathematics achievement.

Butte School District

School District Overview

Butte School District 1 is a rural district in Butte, Montana. The district includes 6 elementary schools and 1 secondary with a current enrollment of over 2,900 students in Grades Pre-K–8. The professional staff includes 184 teachers, with approximately 50 support personnel and administrators.

Butte School District 1 is eligible for Title 1 assistance. Ninety-five percent of the students are Caucasian, and the remaining student body consists of students identified as Hispanic (2%), Asian (2%), or other (1%). Seventeen percent of the students qualify for free/reduced lunch and 9% percent of the students have an Individualized Education Program (IEP).

Montana Criterion-Referenced Test

The Montana Criterion-Referenced Test (CRT) is taken by all students in Grades 3–8 and 10, in the spring. The test includes assessment of reading, mathematics, and science. The test focuses on how well students have acquired the skills and knowledge described in the state Content Standards.

At Grades 3–5, the math Montana CRT is comprised of multiple choice, short-constructed response, and extended response items covering seven standards: Problem Solving, Number & Operations, Algebraic Concepts, Geometry, Measurement, Data Analysis, Probability & Statistics, Patterns, Relations & Functions.

The Montana CRT test scores are categorized into one of four levels of mastery including Level 1 (Advanced), Level 2 (Proficient), Level 3 (Nearing Proficient), and Level 4 (Novice). Students scoring at Advanced and Proficient are said to excel in the state content standards.

Student Success with *GO Math!* © 2012

To determine if HMH *GO Math!* had an impact on learning, NY State mathematics test scores from the Spring of 2012, prior to using the program and the Spring of 2013, after one year of usage, were obtained for Grades 3, 4, 5, and 6. These findings are presented in Figure 1.

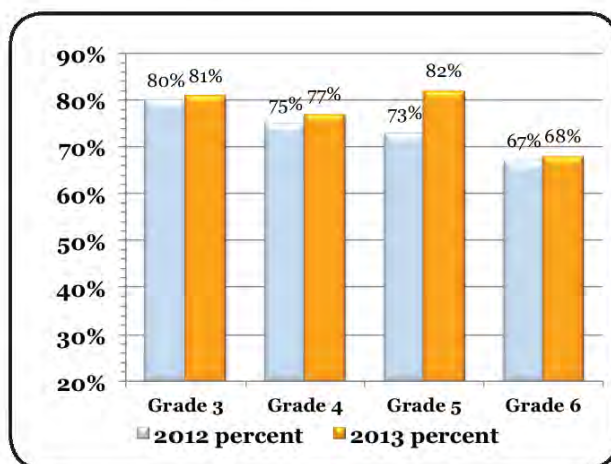


Figure 1

Percent of Students Proficient & Advanced

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Proficient or higher was greater in 2013, after using *GO Math!*, than the previous year with an average gain of over 4%.

Conclusion

This one-year examination of the implementation of *GO Math!* © 2012 at Butte School District 1 revealed that the program was associated with increased mathematics achievement.

Carroll Community School District

School District Overview

Carroll Community School District is a rural school district in Carroll, Iowa. The district includes 3 elementary schools and 3 secondary schools with a current enrollment of over 1,800 students in Grades Pre-K–12. The professional staff includes 119 teachers, with approximately 138 support personnel and administrators.

Ninety percent of the students are Caucasian, and the remaining student body consists of students identified as Hispanic (5%), Black (2%), or other (3%). Thirty-two percent of the students qualify for free/reduced lunch and 12% of the students have an Individualized Education Program (IEP).

Iowa Assessments

Iowa Assessments™ is taken by all public school students, Grades 3–8 and 11, in the spring. The test covers reading, mathematics, and science. The test measures how well students acquire the skills and knowledge described in the Iowa Content Standards.

At Grades 3–5, the math *Iowa Assessments* is comprised of multiple choice and open-ended covering five domains: Number Sense and Operations, Algebraic Patterns and Connections, Data Analysis/Probability/Statistics, Geometry, and Measurement.

In Iowa, students' test scores can be categorized into one of three levels of mastery including Level 1 (Not Proficient), Level 2 (Proficient), and Level 3 (Advanced). Students scoring at the top two levels (Proficient and Advanced) are classified as excelling on the Iowa math standards.

Student Success with *GO Math!* © 2012

To determine if HMH *GO Math!* had an impact on learning, students' *Iowa Assessments* mathematics test scores from the Spring of 2013, prior to using the program and the Spring of 2014, after one year of usage were obtained for Grades 3, 4, and 5. These findings are presented in Figure 1.

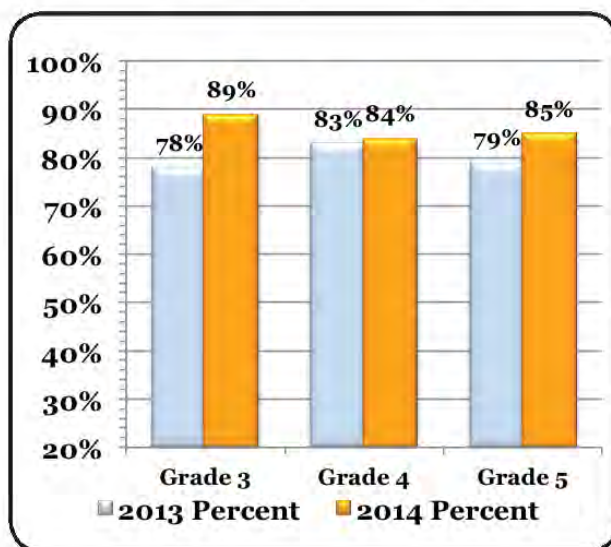


Figure 1

Percent of Students Proficient and Advanced

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Advanced or higher was greater in 2014, after using *GO Math!*, than the previous year with an average gain of over 6%.

Conclusion

This one-year examination of the implementation of *GO Math!* © 2012 at Carroll Community School District revealed that the program was associated with increased mathematics achievement.

DC Public Schools

School District Overview

DC Public Schools is an urban school district in Washington, DC. The district includes 60 elementary schools and 26 secondary schools with a current enrollment of over 46,000 students in Grades Pre-K–12. The professional staff includes 3,500 teachers, with approximately 3,600 support personnel and administrators.

DC Public Schools is eligible for Title I assistance. Sixty-seven percent of the students are Black, and the remaining student body consists of students identified as Hispanic (17%), Caucasian (12%), or other (4%). Seventy-six percent of the students qualify for free/reduced lunch and 16% of the students have an Individualized Education Program (IEP).

District of Columbia Comprehensive Assessment System

District of Columbia Comprehensive Assessment System (DC-CAS) is taken by all public school students, Grades 3–8 and 10, in the spring. The test covers reading, mathematics, and science and is administered to students in Grades 5 and 8. The test measures how well students acquire the skills and knowledge described in the DC Content Standards.

At Grades 3–5, the math DC-CAS is comprised of multiple choice and open-ended items covering five domains: Number Sense and Operations; Patterns, Relations, and Algebra; Geometry; Measurement; and Data Analysis, Statistics, and Probability.

In DC, students' test scores can be categorized into one of four levels of mastery including Level 1 (Below Basic), Level 2 (Basic), Level 3 (Proficient), and Level 4 (Advanced). Students scoring at the top two levels (Proficient and Advanced) are classified as excelling on the DC math standards.

Student Success with *GO Math!* © 2012

To determine if HMH *GO Math!* had an impact on learning, students' DC-CAS mathematics test scores from the Spring of 2013, prior to using the program and the Spring of 2014, after one year of usage were obtained for Grades 3, 4, and 5. These findings are presented in Figure 1.

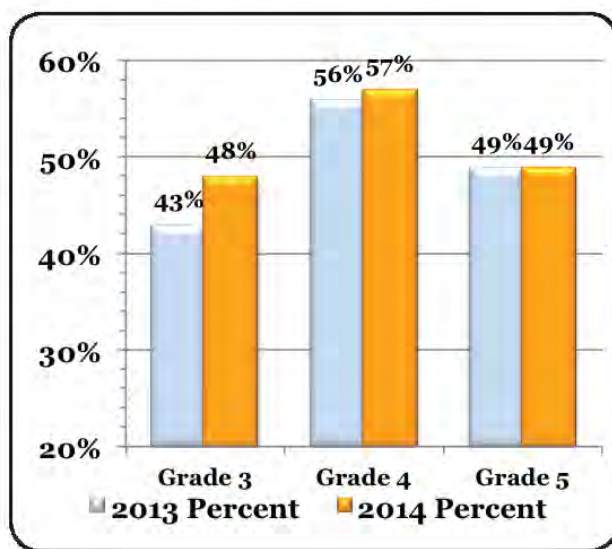


Figure 1
Percent of Students Proficient & Advanced

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Exceeds or higher was greater in 2014, after using *GO Math!*, than the previous year with an average gain of over 2%.

Conclusion

This one-year examination of the implementation of *GO Math!* © 2012 at DC Public Schools revealed that the program was associated with increased mathematics achievement.

Franklin Township Public School District

School District Overview

Franklin Township Public School District is a suburban district in Somerset, New Jersey with 6 elementary schools and 3 secondary schools. The district has a current enrollment of over 7,600 students in Grades K–12. The professional staff includes 641 teachers, with approximately 552 support personnel and administrators.

Thirty-eight percent of the students are Black and the remaining student body consists of students identified as Hispanic (27%), Asian (18%), Caucasian (15%), or other (2%). Forty-seven percent of the students are eligible for free/reduced lunch and 6% percent of the students have an Individualized Education Program (IEP).

New Jersey Assessment of Skills and Knowledge

New Jersey Assessment of Skills and Knowledge (NJ ASK) is a test taken by Grade 3–8 public school students in the spring. The test covers subjects such as ELA, mathematics, and science and measures performance based on the Core Curriculum Content Standards.

At Grades 3–5, the math NJ ASK is comprised of multiple choice, short-answer, and open response items covering four domains equally distributed across the tests: Number & Numerical Operations, Geometry & Measurement, Patterns & Algebra, and Data Analysis & Probability.

In New Jersey, students' test scores can be categorized into one of three levels of mastery: Level 1: Partially Proficient, Level 2: Proficient, and Level 3: Advanced Proficient. Students scoring at Advanced Proficient and Proficient are said to have mastered the New Jersey learning standards.

Student Success with *GO Math!* © 2012

To determine if HMH *GO Math!* had an impact on learning, student NJ ASK mathematics test scores from the Spring of 2013, prior to using the program and the Spring of 2014, after one year of usage were obtained for Grades 3, 4, 5, and 6. These findings are presented in Figure 1.

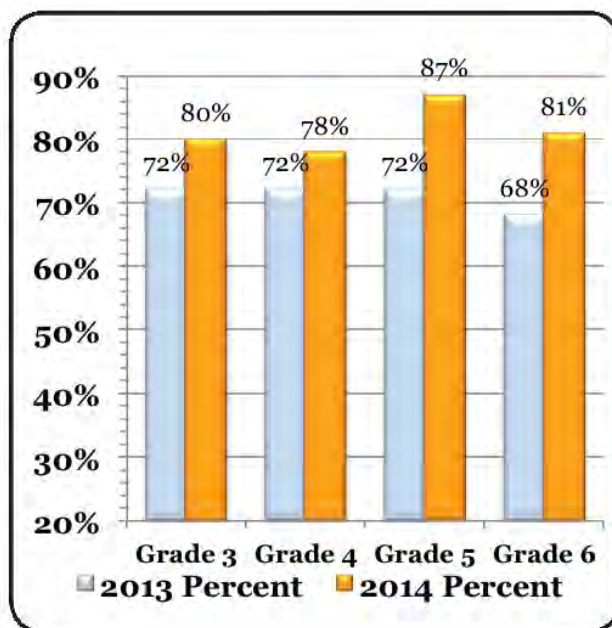


Figure 1
Percent of Students
Proficient/Advanced Proficient

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Proficient or higher was greater in 2014, after using *GO Math!*, than the previous year with an average gain of over 10%.

Conclusion

This one-year examination of the implementation of *GO Math!* © 2012 at Franklin Township Public School District revealed that the program was associated with increased mathematics achievement.

Freeport Public School District

School District Overview

Freeport Public School District is a suburban district in Freeport, New York. The district includes 6 elementary schools and 1 secondary school with a current enrollment of over 6,700 students in Grades Pre-K–12. The professional staff includes 506 teachers, with approximately 520 support personnel and administrators.

Sixty percent of the students in Freeport School District are Hispanic, and the remaining student body consists of students identified as Black (29%), Caucasian (7%), or other (4%). Sixty-four percent of the students qualify for free/reduced lunch and 14% percent of the students have an Individualized Education Program (IEP).

New York State Testing Program

The annual New York State Test is taken by all students in Grades 3–8, in the spring. The test includes assessment of ELA, math, and science. The test focuses on how well students have acquired the skills and knowledge described in the state Common Core Learning Standards.

At Grades 3–5, the math New York State Test is comprised of multiple choice, short-constructed response, and extended response items covering five domains equally distributed across the tests: Geometry, Measurement and Data, Numbers and Operations in Base Ten, Number and Operations-Fractions, and Operations and Algebraic Thinking.

The New York State Test scores are categorized into one of four levels of mastery including Level 1 (Well below Proficient), Level 2 (Below Proficient), Level 3 (Proficient), and Level 4 (Above Proficient). Students scoring at Levels 3 and 4 are said to excel and demonstrate knowledge, skills, and practices embodied by the New York State P–12 Common Core Learning Standards for Mathematics.

Student Success with *GO Math!* © 2012

To determine if HMH *GO Math!* had an impact on learning, NY State Math Test scores from the Spring of 2013, prior to using the program and the Spring of 2014, after one year of usage, were obtained for Grades 3, 4, and 5. These findings are presented in Figure 1.

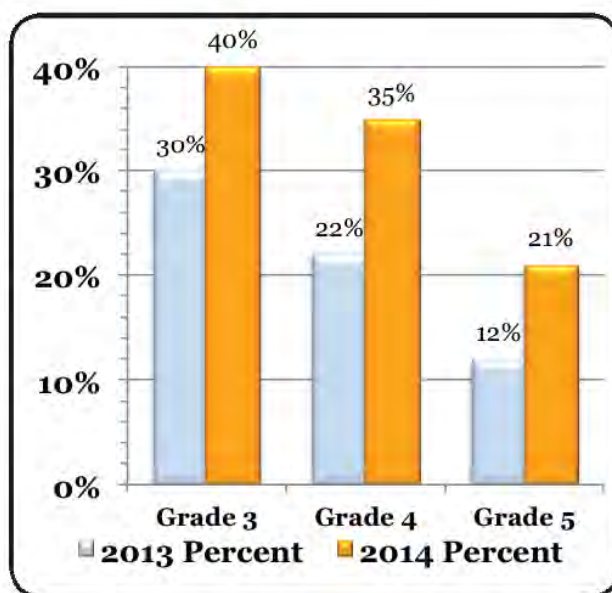


Figure 1
Percent of Students Level 3/Level 4

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Level 3 or higher was greater in 2014, after using *GO Math!*, than the previous year with an average gain of over 11%.

Conclusion

This one-year examination of the implementation of *GO Math!* © 2012 at Freeport Public School District revealed that the program was associated with increased mathematics achievement.

Greenup County School District

School District Overview

Greenup County School District is a suburban district in Greenup, Kentucky. The district includes 4 elementary schools and 3 secondary with a current enrollment of over 2,800 students in Grades K–12. The professional staff includes 185 teachers, with approximately 260 support personnel and administrators.

Greenup County School District is eligible for Title I assistance. Ninety-seven percent of the students are Caucasian, and the remaining student body consists of students identified as other (3%). Sixty-six percent of the students qualify for free/reduced lunch and 13% percent of the students have an Individualized Education Program (IEP).

Kentucky Performance Rating for Educational Progress

The Kentucky Performance Rating for Educational Progress (K-PREP) is taken by all students in Grades 3–8, 10, and 11 in the spring. The test includes assessment of reading, mathematics, social studies, and science. The test focuses on how well students have acquired the skills and knowledge described in the state standards.

At Grades 3–5, the math K-PREP is comprised of multiple choice, short-constructed response, and extended response items covering five domains: Operations and Algebraic Thinking, Number and Operations in Base Ten, Number and Operations—Fractions, Measurement and Data, and Geometry.

The Kentucky state test scores are categorized into one of four levels of mastery including Level 1 (Novice), Level 2 (Apprentice), Level 3 (Proficient), and Level 4 (Distinguished). Students scoring at Proficient or above are excelling.

Student Success with GO Math! © 2012

To determine if HMH **GO Math!** had an impact on learning, K-PREP mathematics test scores from the Spring of 2013, prior to using the program and the Spring of 2014, after one year of usage, were obtained for Grades 3, 4, and 5. These findings are presented in Figure 1.

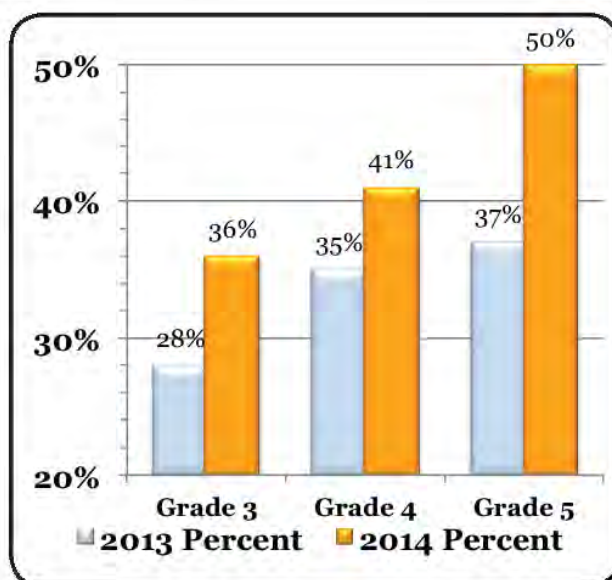


Figure 1
Percent of Students Proficient and Distinguished

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Proficient or higher was greater in 2014, after using **GO Math!**, than the previous year with an average gain of over 9%.

Conclusion

This one-year examination of the implementation of **GO Math!** © 2012 at Greenup County School District revealed that the program was associated with increased mathematics achievement.

Holyoke Public School District

School District Overview

Holyoke Public School District is an urban district in Holyoke, Massachusetts with 9 elementary schools and 2 secondary schools. The district has a current enrollment of over 5,500 students in Grades Pre-K–12. The professional staff includes 503 teachers, with approximately 497 support personnel and administrators.

Holyoke Public School District is eligible for Title I assistance. Seventy-nine percent of the students are Hispanic and the remaining student body consists of students identified as Caucasian (16%) or other (5%). Fourteen percent of the students have an Individualized Education Program (IEP).

Massachusetts Comprehensive Assessment System

Massachusetts Comprehensive Assessment System (MCAS) is a test taken by all public school students in the spring. The test covers subjects such as ELA, mathematics, science, and history. The test measures performance based on the Massachusetts Curriculum Framework Learning Standards.

At Grades 3–5, the math MCAS is comprised of multiple choice, short-answer, and open response items covering four domains equally distributed across the tests: Geometry, Measurements and data, Number and Operations in Base Ten and Fractions, and Algebraic Thinking.

In Massachusetts, students' test scores can be categorized into one of four levels of mastery: Level 1: Advanced, Level 2: Proficient, Level 3: Needs Improvement, and Level 4: Warning. Students scoring at Advanced and Proficient are said to have mastered the Massachusetts Learning Standards.

Student Success with *GO Math!* © 2012

To determine if HMH *GO Math!* had an impact on learning, student MCAS mathematics test scores from the Spring of 2012, prior to using the program and the Spring of 2013, after one year of usage were obtained for Grades 3, 4, 5, and 6. These findings are presented in Figure 1.

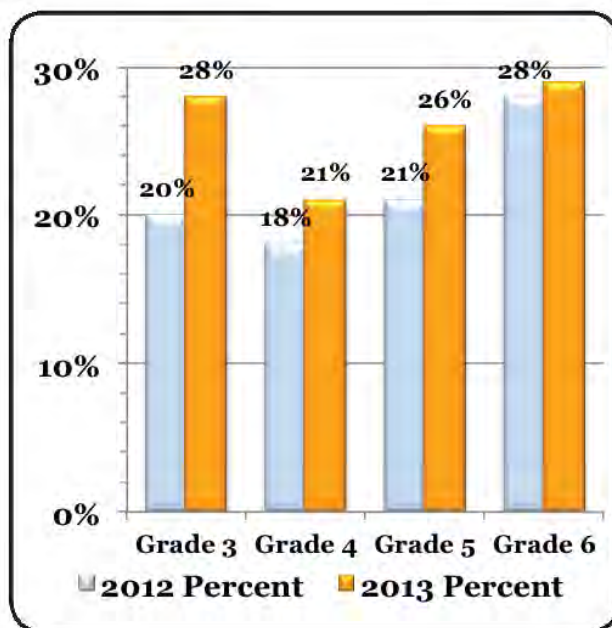


Figure 1

Percent of Students Proficient/Advanced

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Proficient or higher was greater in 2013, after using *GO Math!* than the previous year with an average gain of over 5%.

Conclusion

This one-year examination of the implementation of *GO Math!* © 2012 at Holyoke Public School District revealed that the program was associated with increased mathematics achievement.

Limestone County School District

School District Overview

Limestone County School District is a rural school district in Athens, Alabama. The district includes 6 elementary schools and 7 secondary schools with a current enrollment of over 8,900 students in Grades Pre-K–12. The professional staff includes 600 teachers.

Eighty-two percent of the students are Caucasian, and the remaining student body consists of students identified as Black (9%), Hispanic (7%), or other (2%). Forty-seven percent of the students qualify for free/reduced lunch and 10% of the students have an Individualized Education Program (IEP).

Alabama Reading and Mathematics Test

Alabama Reading and Mathematics Test (ARMT) is taken by all public school students, Grades 3–8, in the spring. The test covers reading and mathematics. The test measures how well students acquire the skills and knowledge described in the Alabama Content Standards.

At Grades 3–5, students must take Stanford 10 Mathematics procedures, mathematics Problem Solving, and the ARMT Part 2 Mathematics subtest to get an ARMT mathematics score. The math ARMT is comprised of multiple choice and open-ended items covering five domains: Numbers and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability.

In Alabama, students' test scores can be categorized into one of four levels of mastery including Level 1 (Does not meet), Level 2 (Partially meets), Level 3 (Meets), and Level 4 (Exceeds). Students scoring at the top two levels (Meets and Exceeds) are classified as excelling on the Alabama math standards.

Student Success with *GO Math!* © 2012

To determine if HMH *GO Math!* had an impact on learning, students' ARMT mathematics test scores from the Spring of 2011, prior to using the program and the Spring of 2012, after one year of usage were obtained for Grades 3, 4, 5, and 6. These findings are presented in Figure 1.

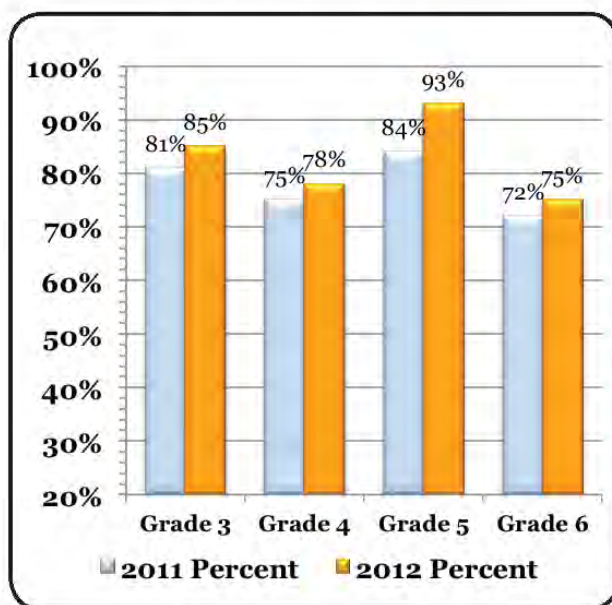


Figure 1

Percent of Students Meets and Exceeds

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Exceeds or higher was greater in 2012, after using *GO Math!*, than the previous year with an average gain of over 4%.

Conclusion

This one-year examination of the implementation of *GO Math!* © 2012 at Limestone County School District revealed that the program was associated with increased mathematics achievement.

Marblehead Public School District

School District Overview

Marblehead Public School District is a suburban district in Marblehead, Massachusetts with 5 elementary schools and 2 secondary schools. The district has a current enrollment of over 3,200 students in Grades Pre-K–12. The professional staff includes 230 teachers, with approximately 215 support personnel and administrators.

Marblehead Public School District is eligible for Title I assistance. Eighty-seven percent of the students are Caucasian and the remaining student body consists of students identified as Hispanic (6%), Black (3%), or other (4%). Eighteen percent of the students have an Individualized Education Program (IEP).

Massachusetts Comprehensive Assessment System

Massachusetts Comprehensive Assessment System (MCAS) is a test taken by all public school students in the spring. The test covers subjects such as ELA, mathematics, science, and history. The test measures performance based on the Massachusetts Curriculum Framework Learning Standards.

At Grades 3–5, the math MCAS is comprised of multiple choice, short-answer, and open response items covering four domains equally distributed across the tests: Geometry, Measurements and data, Number and Operations in Base Ten and Fractions, and Algebraic Thinking.

In Massachusetts, students' test scores can be categorized into one of four levels of mastery: Level 1: Advanced, Level 2: Proficient, Level 3: Needs Improvement, and Level 4: Warning. Students scoring at Advanced and Proficient are said to have mastered the Massachusetts Learning Standards.

Student Success with *GO Math!* © 2012

To determine if HMH *GO Math!* had an impact on learning, student MCAS mathematics test scores from the Spring of 2013, prior to using the program and the Spring of 2014, after one year of usage were obtained for Grades 3, 4, and 5. These findings are presented in Figure 1.

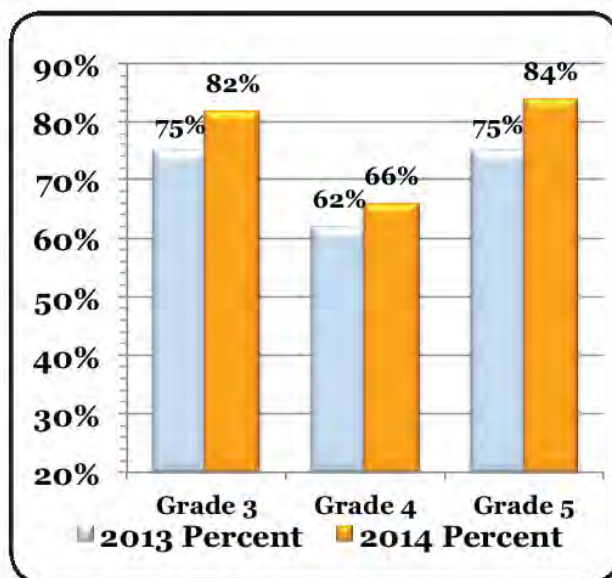


Figure 1
Percent of Students Proficient/Advanced

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Proficient or higher was greater in 2014, after using *GO Math!*, than the previous year with an average gain of over 7%.

Conclusion

This one-year examination of the implementation of *GO Math!* © 2012 at Marblehead Public School District revealed that the program was associated with increased mathematics achievement.

Quincy Public School District

School District Overview

Quincy Public School District is a suburban district in Quincy, Massachusetts with 12 elementary schools and 7 secondary schools. The district has a current enrollment of over 9,300 students in Grades Pre-K–12. The professional staff includes 655 teachers, with approximately 376 support personnel and administrators.

Quincy Public School District is eligible for Title I assistance. Forty-Nine percent of the students are Caucasian and the remaining student body consists of students identified as Asian (36%), Black (5%), Hispanic (5%), or other (5%). Sixteen percent of the students have an Individualized Education Program (IEP).

Massachusetts Comprehensive Assessment System

Massachusetts Comprehensive Assessment System (MCAS) is a test taken by all public school students in the spring. The test covers subjects such as ELA, mathematics, science, and history. The test measures performance based on the Massachusetts Curriculum Framework Learning Standards.

At Grades 3–5, the math MCAS is comprised of multiple choice, short-answer, and open response items covering four domains equally distributed across the tests: Geometry, Measurements and data, Number and Operations in Base Ten and Fractions, and Algebraic Thinking.

In Massachusetts, students' test scores can be categorized into one of four levels of mastery: Level 1: Advanced, Level 2: Proficient, Level 3: Needs Improvement, and Level 4: Warning. Students scoring at Advanced and Proficient are said to have mastered the Massachusetts Learning Standards.

Student Success with *GO Math!* © 2012

To determine if HMH *GO Math!* had an impact on learning, student MCAS mathematics test scores from the Spring of 2012, prior to using the program and the Spring of 2013, after one year of usage were obtained for Grades 3, 4, 5, and 6. These findings are presented in Figure 1.

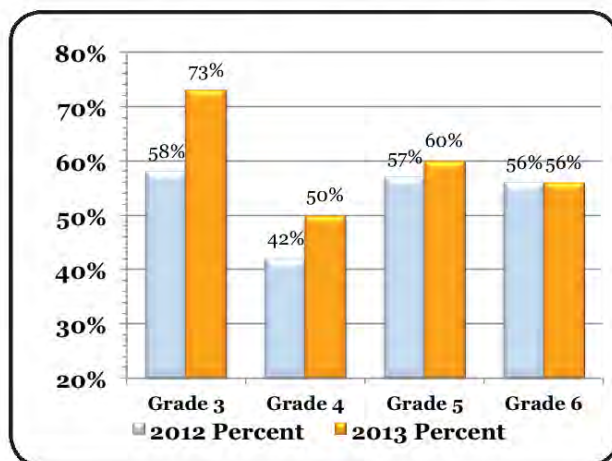


Figure 1

Percent of Students Proficient/Advanced

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Proficient or higher was greater in 2013, after using *GO Math!*, than the previous year with an average gain of over 6%.

Conclusion

This one-year examination of the implementation of *GO Math!* © 2012 at Quincy Public School District revealed that the program was associated with increased mathematics achievement.

Scott County School District

School District Overview

Scott County School District is a rural district in Georgetown, Kentucky. The district includes 8 elementary schools and 6 secondary with a current enrollment of over 8,400 students in Grades K–12. The professional staff includes 486 teachers, with approximately 594 support personnel and administrators.

Scott County School District is eligible for Title I assistance. Eighty-three percent of the students are Caucasian, and the remaining student body consists of students identified as Hispanic (7%), Black (7%), or other (3%). Forty-two percent of the students qualify for free/reduced lunch and 13% percent of the students have an Individualized Education Program (IEP).

Kentucky Performance Rating for Educational Progress

The Kentucky Performance Rating for Educational Progress (K-PREP) is taken by all students in Grades 3–8, 10, and 11 in the spring. The test includes assessment of reading, mathematics, social studies, and science. The test focuses on how well students have acquired the skills and knowledge described in the state standards.

At Grades 3–5, the math K-PREP is comprised of multiple choice, short-constructed response, and extended response items covering five domains: Operations and Algebraic Thinking, Number and Operations in Base Ten, Number and Operations—Fractions, Measurement and Data, and Geometry.

The Kentucky state test scores are categorized into one of four levels of mastery including Level 1 (Novice), Level 2 (Apprentice), Level 3 (Proficient), and Level 4 (Distinguished). Students scoring at Proficient or above are excelling.

Student Success with **GO Math!** © 2012

To determine if HMH **GO Math!** had an impact on learning, K-PREP mathematics test scores from the Spring of 2013, prior to using the program and the Spring of 2014, after one year of usage, were obtained for Grades 3, 4, and 5. These findings are presented in Figure 1.

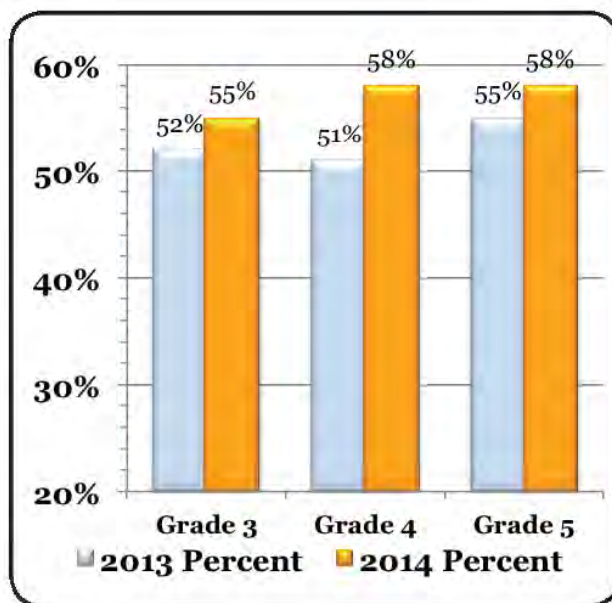


Figure 1
Percent of Students Proficient and Distinguished

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Proficient or higher was greater in 2014, after using **GO Math!**, than the previous year with an average gain of over 5%.

Conclusion

This one-year examination of the implementation of **GO Math!** © 2012 at Scott County School District revealed that the program was associated with increased mathematics achievement.

Streator Elementary School District

School District Overview

Streator Elementary School District 44 is a rural district in Streator, Illinois with 3 elementary schools and 1 secondary school. The district has a current enrollment of over 1,800 students in Grades Pre-K–8. The professional staff includes 114 teachers, with approximately 107 support personnel and administrators.

Streator Elementary School District 44 is eligible for Title I assistance. Seventy-one percent of the students are Caucasian and the remaining student body consists of students identified as Hispanic (19%), Black (4%), or other (6%). Sixty-nine percent of the students qualify for free or reduced lunch and 22% have an Individualized Education Program (IEP).

Illinois Standards Achievement Test

Illinois Standards Achievement Test (ISAT) is administered to students in Grades 3–8 in the spring. The test covers subjects such as reading, mathematics, and science. The test measures performance based on the Illinois Curriculum Framework Learning Standards.

At Grades 3–5, the math ISAT is comprised of multiple choice, short-answer, and open response items covering five domains: Geometry, Measurements and data, Number and Operations in Base Ten, Numbers and Operations in Fractions, and Operations and Algebraic Thinking.

In Illinois, students' test scores can be categorized into one of four levels of mastery: Level 1: Academic Warning, Level 2: Below Standards, Level 3: Meets Standards, and Level 4: Exceeds Standards. Students scoring at Exceeds and Meets are said to have mastered the Illinois Learning Standards.

Student Success with **GO Math!** © 2012

To determine if HMH **GO Math!** had an impact on learning, student ISAT mathematics test scores from the Spring of 2013, prior to using the program and the Spring of 2014, after one year of usage were obtained for Grades 3, 4, 5, and 6. These findings are presented in Figure 1.

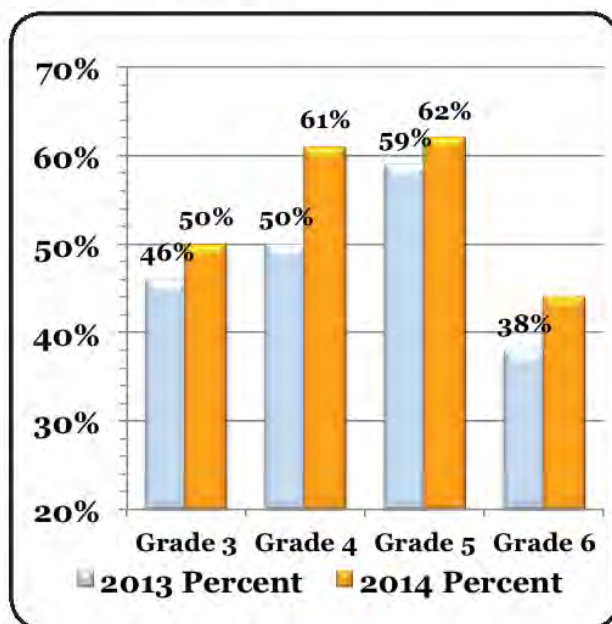


Figure 1

Percent of Students Meets / Exceeds

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Meets Standards or higher was greater in 2014, after using **GO Math!**, than the previous year with an average gain of over 6%.

Conclusion

This one-year examination of the implementation of **GO Math!** © 2012 at Streator Elementary School District 44 revealed that the program was associated with increased mathematics achievement.

Tewksbury Public School District

School District Overview

Tewksbury Public School District is a suburban district in Tewksbury, Massachusetts with 5 elementary schools and 2 secondary schools. The district has a current enrollment of over 3,800 students in Grades Pre-K–12. The professional staff includes 248 teachers, with approximately 184 support personnel and administrators.

Tewksbury Public School is eligible for Title I assistance. Ninety-two percent of the students are Caucasian and the remaining student body consists of students identified as Asian (3%), Hispanic (2%), or other (3%). Ten percent of the students are eligible for free/reduced lunch and 17% have an Individualized Education Program (IEP).

Massachusetts Comprehensive Assessment System

Massachusetts Comprehensive Assessment System (MCAS) is a test taken by all public school students in the spring. The test covers subjects such as ELA, mathematics, science, and history. The test measures performance based on the Massachusetts Curriculum Framework Learning Standards.

At Grades 3–5, the math MCAS is comprised of multiple choice, short-answer, and open response items covering four domains equally distributed across the tests: Geometry, Measurements and data, Number and Operations in Base Ten and Fractions, and Algebraic Thinking.

In Massachusetts, students' test scores can be categorized into one of four levels of mastery: Level 1: Advanced, Level 2: Proficient, Level 3: Needs Improvement, and Level 4: Warning. Students scoring at Advanced and Proficient are said to have mastered the Massachusetts Learning Standards.

Student Success with *GO Math!* © 2012

To determine if HMH *GO Math!* had an impact on learning, student MCAS mathematics test scores from the Spring of 2011, prior to using the program and the Spring of 2012, after one year of usage were obtained for Grades 3 and 4. These findings are presented in Figure 1.

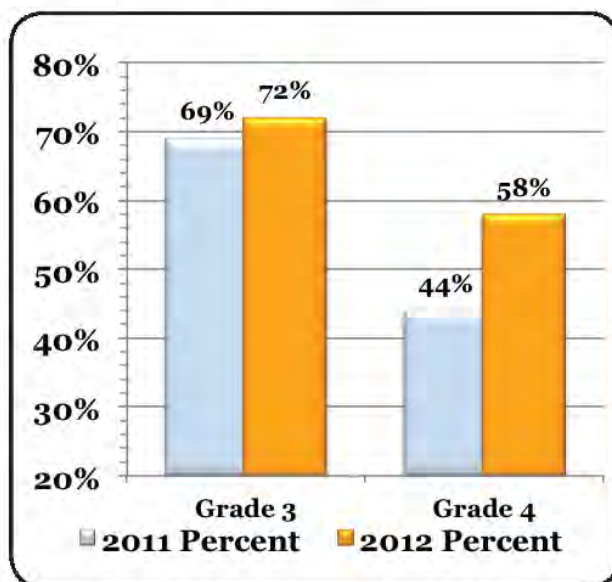


Figure 1
Percent of Students Proficient/Advanced

The comparison of achievement scores revealed that at all grade levels examined, the percentage of students scoring at Proficient or higher was greater in 2013, after using *GO Math!*, than the previous year with an average gain of over 15%.

Conclusion

This one-year examination of the implementation of *GO Math!* © 2012 at Tewksbury Public School District revealed that the program was associated with increased mathematics achievement.

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NOTES

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